Complex eigenvalues

Find the complex eigenvalues and eigenvectors for

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$$

Announcements Nov 15

- Masks $\rightsquigarrow$ Thank you!
- WeBWorK 5.4 & 5.5 due Tue @ midnight
- Office hrs: Tue 4-5 Teams + Thu 1-2 Skiles courtyard/Teams + Pop-ups
- Review sessions: Prof. M Mon and Wed 4:30–5:15 Howey L1
- Review session: Joe Cochran Tue 7:15–9:15 Skiles 371(???)
- Midterm 3 Nov 17 8–9:15 on Teams, Sec. 3.5–5.5

- Many TA office hours listed on Canvas
- PLUS sessions: Tue 6–7 GT Connector, Thu 6–7 BlueJeans
- Indoor Math Lab: Mon–Thu 11–6, Fri 11–3 Clough 246 + 252
- Outdoor Math Lab: Tue–Thu 2–4 Skiles Courtyard
- Virtual Math Lab `https://tutoring.gatech.edu/drop-in/`
- Section M web site: Google "Dan Margalit math", click on 1553
  - future blank slides, past lecture slides, advice
- Old exams: Google "Dan Margalit math", click on Teaching
- Tutoring: `http://tutoring.gatech.edu/tutoring`
- Counseling center: `https://counseling.gatech.edu`
- Use Piazza for general questions
- You can do it!

Section 5.5

Complex Eigenvalues

Outline of Section 5.5

- Rotation matrices have no eigenvectors
- Crash course in complex numbers
- Finding complex eigenvectors and eigenvalues
- Complex eigenvalues correspond to rotations + dilations

Demo Demo

Section 5.6

Stochastic Matrices (and Google!)

Outline of Section 5.6

- Stochastic matrices and applications
- The steady state of a stochastic matrix
- Important web pages

Stochastic matrices

A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.

Examples:

$$\begin{pmatrix} 1/4 & 3/5 \\ 3/4 & 2/5 \end{pmatrix} \quad \begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & 1 & 1/2 \\ 1/2 & 0 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix}$$

Application: Rental Cars (or Redbox...)

Say your car rental company has 3 locations. Make a matrix whose ij entry is the fraction of cars at location j that end up at location i. For example,

.3 of cars from 1 end up at 1

.4 of cars from 1 end up at 3

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix} = A$$

Note the columns sum to 1. Why?

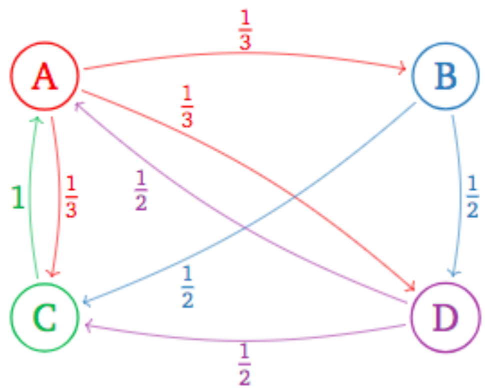
All cars end up somewhere.

If there are 100 cars at each location on the first day, and every car gets rented, how many cars are at each location on the second day? third day? nth day?

$$\underbrace{A\begin{pmatrix}100\\100\\100\end{pmatrix}}_{\text{after 1 day}} = \begin{pmatrix}120\\100\\80\end{pmatrix} \qquad A\begin{pmatrix}120\\100\\80\end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix} \text{ after 2 day}$$

Application: Web pages

Make a matrix whose ij entry is the fraction of (randomly surfing) web surfers at page j that end up at page i. If page i has N links then the ij-entry is either 0 or $1/N$.



$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Cole: C 3 in

Stefano: A 3 out

Barry: C goes all to A

Patrick: largest row.

Which web page seems most important?

Properties of stochastic matrices

Let A be a stochastic matrix.

Fact 1. One of the eigenvalues of A is 1 and all other eigenvalues have absolute value at most 1.

Why?

$$A = \begin{pmatrix} 1/4 & 3/5 \\ 3/4 & 2/5 \end{pmatrix} \rightsquigarrow A^T = \begin{pmatrix} 1/4 & 3/4 \\ 3/5 & 2/5 \end{pmatrix}$$

A^T has 1 as eigenvalue: $\begin{pmatrix} 1/4 & 3/4 \\ 3/5 & 2/5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

But A^T & A have same eigenvalues!

(Second part is a little harder).

Positive stochastic matrices

Let A be a positive stochastic matrix, meaning all entries are positive. (Before, we allowed zeros).

Fact 2. The 1-eigenspace of A is 1-dimensional; it has a positive eigenvector.

The unique such eigenvector with entries adding to 1 is called the steady state vector.

Example. If $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is a 1-eigenvector, what's the steady state vector?

divide by sum of entries:

$$\begin{pmatrix} 1/6 \\ 2/6 \\ 3/6 \end{pmatrix} = \begin{pmatrix} 1/6 \\ 1/3 \\ 1/2 \end{pmatrix}$$

another 1-eigenvec.

Example

Find the steady state vector.

$$A = \begin{pmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix}$$

positive ✓

stochastic ✓

1 - eigenvector: $\begin{pmatrix} -3/4 & 3/4 \\ 3/4 & -3/4 \end{pmatrix} \xrightarrow[\text{\& scale by } -4/3]{\text{trick 1}} \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \xrightarrow{\text{trick 2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Steady state vector: $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$.

More about positive stochastic matrices

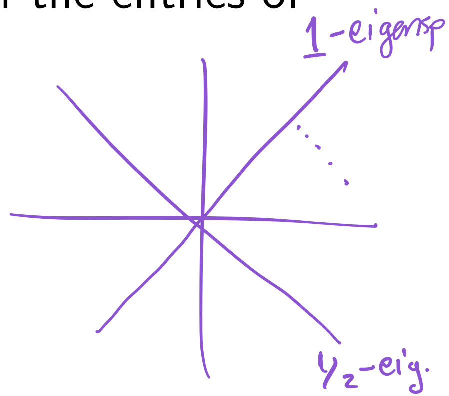
Let A be a positive stochastic matrix, meaning all entries are positive.

Fact 3. Under iteration, all nonzero vectors approach a multiple of the steady state vector. The multiple is the sum of the entries of the original vector.

Demo

On last slide:

$$\text{SSV} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$



$$A^n \begin{pmatrix} 30 \\ 42 \end{pmatrix} \longrightarrow \begin{pmatrix} 36 \\ 36 \end{pmatrix} = 72 \cdot \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

sum of entries is 72.

The last fact tells us how to distribute rental cars, and also tells us the importance of web pages!

Example

To what vector does $A^n \left(\begin{smallmatrix} 1 \\ 9 \end{smallmatrix}\right)$ approach as $n \to \infty$

$$A = \begin{pmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix}$$

$$SSV = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 5 \end{pmatrix} = (9+1) \cdot \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

Application: Rental Cars

The rental car matrix is:

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Its steady state vector is:

1-eigenvector

$$\begin{pmatrix} 7/18 \\ 6/18 \\ 5/18 \end{pmatrix} \approx \begin{pmatrix} .39 \\ .33 \\ .28 \end{pmatrix}$$

If the original distribution of cars is given by $(100, 100, 100)$ what will the distribution of cars be after a very long time?

$$300 \cdot \begin{pmatrix} .39 \\ .33 \\ .28 \end{pmatrix} \approx \begin{pmatrix} 117 \\ 99 \\ 84 \end{pmatrix}$$

Application: Web pages

Damping

.15 .15 $.85 + .15$ $.85\frac{1}{2} + .15$

.15 .15

.15

.15 .15

The web page matrix is:

$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Its steady state vector is approximately

$$\begin{pmatrix} .39 \\ .13 \\ .29 \\ .19 \end{pmatrix}$$

and so the first web page is the most important.

Fine print

There are a couple of problems with the web page matrix as given:

- What happens if there is a web page with no links?
- What if the internet graph is not connected? (Too many 0's)
- How do you find eigenvectors for a huge matrix?

Here are the solutions:

- Make a column with $1/n$ in each entry (the surfer goes to a new page randomly).

  $n \times n$ matrix.
- Let B be the matrix with all entries equal to $1/n$, replace A with

damping factor

$$.85 * A + .15 * B$$

$A^{5000} v$

- Approximate via iteration!

Summary of Section 5.6

- A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.
- Every stochastic matrix has 1 as an eigenvalue, and all other eigenvalues have absolute value at most 1.
- A positive stochastic matrix has 1-dimensional eigenspace and has a positive eigenvector. A positive 1-eigenvector with entries adding to 1 is called a steady state vector.
- For a positive stochastic matrix, all nonzero vectors approach the steady state vector under iteration.
- Steady state vectors tell us the importance of web pages (for example).

Typical Exam Questions 5.6

- Is there a stochastic matrix where the 1-eigenspace has dimension greater than 1?
- Find the steady state vector for this matrix:

$$A = \begin{pmatrix} 1/2 & 1/3 \\ 1/2 & 1/3 \end{pmatrix}$$

  To what vector does $A^n \left(\begin{smallmatrix} 5 \\ 7 \end{smallmatrix} \right)$ approach as $n \to \infty$?
- Find the steady state vector for this matrix:

$$A = \begin{pmatrix} 1/3 & 1/5 & 1/4 \\ 1/3 & 2/5 & 1/2 \\ 1/3 & 2/5 & 1/4 \end{pmatrix}$$

- Make your own internet and see if you can guess which web page is the most important. Check your answer using the method described in this section.

Section 5.1

Eigenvectors and eigenvalues

Eigenvalues geometrically

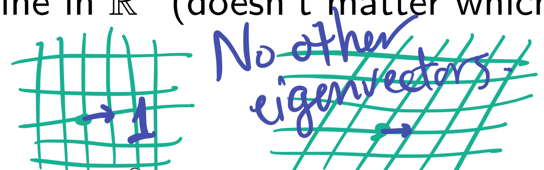
0 eigenvec $Av = 0$

1 eigenvec $Av = v$

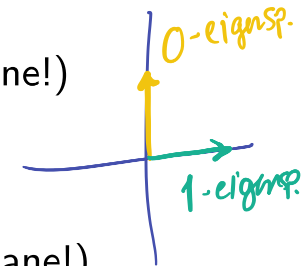
If v is an eigenvector of A then that means v and Av are scalar multiples, i.e. they lie on a line.

Without doing any calculations, find the eigenvectors and eigenvalues of the matrices corresponding to the following linear transformations:

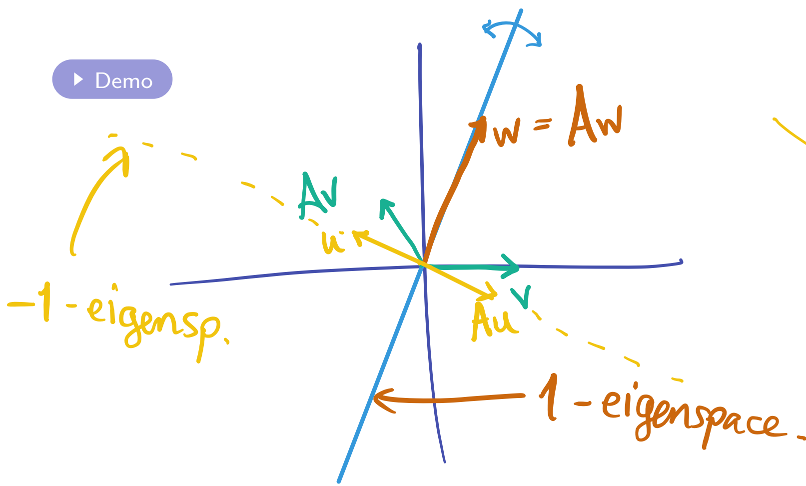
- Reflection about a line in $\mathbb{R}^2$ (doesn't matter which line!)
- Orthogonal projection onto a line in $\mathbb{R}^2$ (doesn't matter which line!)
- Scaling of $\mathbb{R}^2$ by 3
- (Standard) shear of $\mathbb{R}^2$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
- Orthogonal projection to a plane in $\mathbb{R}^3$ (doesn't matter which plane!)

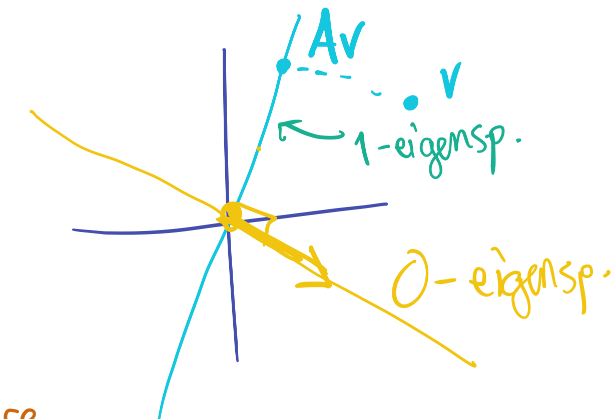


Proj to x-axis



Demo





Typical exam questions 5.1

- Find the 2-eigenvectors for the matrix

$$\begin{pmatrix} 0 & 13 & 12 \\ 1/4 & 0 & 0 \\ 0 & 1/2 & 0 \end{pmatrix}$$

- True or false: The zero vector is an eigenvector for every matrix.
- How many different eigenvalues can there be for an $n \times n$ matrix?
- Consider the reflection of $\mathbb{R}^2$ about the line $y = 7x$. What are the eigenvalues (of the standard matrix)?
- Consider the $\pi/2$ rotation of $\mathbb{R}^3$ about the z-axis. What are the eigenvalues (of the standard matrix)?

Section 5.2

The characteristic polynomial

Characteristic polynomials

3×3 matrices

Find the characteristic polynomial of the following matrix.

$$\begin{pmatrix} 7 & 0 & 3 \\ -3 & 2 & -3 \\ 4 & 2 & 0 \end{pmatrix}$$

Answer: $-\lambda^3 + 9\lambda^2 - 8\lambda$

What are the eigenvalues?

Characteristic polynomials

3×3 matrices

Find the characteristic polynomial of the rabbit population matrix.

$$\begin{pmatrix} 0 & 6 & 8 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Answer:

$$-\lambda^3 + 3\lambda + 2$$

What are the eigenvalues?

Hint: We already know one eigenvalue! Polynomial long division $\rightsquigarrow$

$$(\lambda - 2)(-\lambda^2 - 2\lambda - 1)$$

Don't really need long division: the first and last terms of the quadratic are easy to find; can guess and check the other term.

Characteristic polynomials

3×3 matrices

Find the characteristic polynomial and eigenvalues.

$$\begin{pmatrix} 5 & -2 & 2 \\ 4 & -3 & 4 \\ 4 & -6 & 7 \end{pmatrix}$$

Characteristic polynomial: $-\lambda^3 + 9\lambda^2 - 23\lambda + 15$

This time we don't know any of the roots! We can use the rational root theorem: any integer root of a polynomial with leading coefficient ± 1 divides the constant term.

So we plug in ± 1, ± 3, ± 5, ± 15 into the polynomial and hope for the best. Luckily we find that 1, 3, and 5 are all roots, so we found all the eigenvalues!

If we were less lucky and found only one eigenvalue, we could again use long division like on the last slide.

Characteristic polynomials, trace, and determinant

The trace of a matrix is the sum of the diagonal entries.

The characteristic polynomial always looks like:

$$(-1)^n\lambda^n + (-1)^{n-1}\boxed{\text{trace(A)}}\lambda^{n-1} + \boxed{???}\lambda^{n-2} + \cdots \boxed{???}\lambda + \boxed{\det(\text{A})}$$

So for a 2×2 matrix:

$$\lambda^2 - \text{trace}(A)\lambda + \det(A)$$

And for a 3×3 matrix:

$$-\lambda^3 + \text{trace}(A)\lambda^2 - \boxed{???}\lambda + \det(A)$$

- Sum of eigenvals is trace
- Prod. of eigenvals is det

Typical Exam Questions 5.2

- True or false: Every $n \times n$ matrix has an eigenvalue.
- True or false: Every $n \times n$ matrix has n distinct eigenvalues.
- True or false: The nullity of $A - \lambda I$ is the dimension of the λ-eigenspace.
- What are the eigenvalues for the standard matrix for a reflection?
- What are the eigenvalues and eigenvectors for the $n \times n$ zero matrix?
- Find the eigenvalues of the following matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -5 & 0 \\ 1 & 8 & 0 \end{pmatrix}$$

- Find the eigenvalues of the following matrix.

$$\begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & 2 \end{pmatrix}$$

Hint: All of the eigenvalues are integers. Use the rational root theorem to guess one of the eigenvalues, and then factor out a linear term.

Section 5.4

Diagonalization

Fibonacci numbers

for fun.

Diagonalize the matrix.

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Eigenvalues are φ *&* $-1/\varphi$*, with eigenvectors* $(\varphi, 1)$ *&* $(-1/\varphi, 1)$

What does this tell us about Fibonacci numbers? How quickly do they grow? What is the ratio between consecutive Fibonacci numbers?

Use this to give a formula for the nth Fibonacci number

More Examples

Diagonalize if possible.

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

Hint: the eigenvalues (with multiplicity) are $3, -1, 1$ and $2, 2, 1$

Poll

Poll

Which are diagonalizable?

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

More rabbits

Which ones are diagonalizable?

$$\begin{pmatrix} 0 & 4 \\ \frac{1}{2} & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 6 & 8 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 4 & 4 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Hint: the characteristic polynomials are $-\lambda^3 + 3\lambda + 2$ *and* $-\lambda^3 + 2\lambda + 1$ *and both have rational roots.*

Interpret all of these as rabbit matrices. What can you say about the rabbit populations?

Typical Exam Questions 5.4

- True or False. If A is a 3×3 matrix with eigenvalues 0, 1, and 2, then A is diagonalizable.
- True or False. It is possible for an eigenspace to be 0-dimensional.
- True or False. Diagonalizable matrices are invertible.
- True or False. Diagonal matrices are diagonalizable.
- True or False. Upper triangular matrices are diagonalizable.
- Find the 100th power of $\left(\begin{smallmatrix} 2 & 6 \\ 0 & -1 \end{smallmatrix}\right)$.
- For each of the following matrices, diagonalize or show they are not diagonalizable:

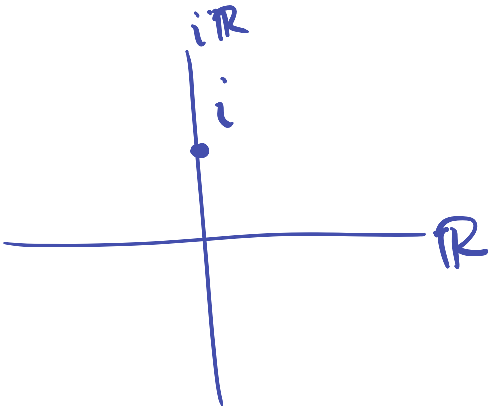
$$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

Section 5.5

Complex Eigenvalues

Complex eigenvalues

Find the complex eigenvalues and eigenvectors for



$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

CCW by $\pi/2$

eigenvals $\pm i$.

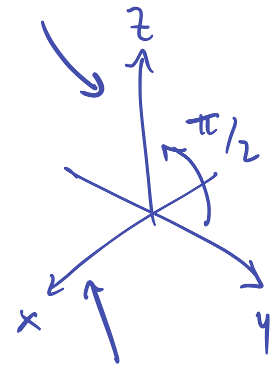
i- eigenvec: $\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \xrightarrow{\text{trick 1}} \begin{pmatrix} -i & -1 \\ 0 & 0 \end{pmatrix} \xrightarrow[2]{\text{trick}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

<u>Trick 3</u> $\bar{i}$ eigenvec is $\begin{pmatrix} 1 \\ -i \end{pmatrix}$

$-i$ eigenvec is $\begin{pmatrix} 1 \\ i \end{pmatrix}$

Complex eigenvalues

Guess: $\pm 2i$ - eigensp.



Find the complex eigenvalues and eigenvectors for

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \boxed{0} & \boxed{-2} \\ 0 & \boxed{2} & \boxed{0} \end{pmatrix}$$

eigenvals

$$\det \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & -2 \\ 0 & 2 & -\lambda \end{pmatrix} = (1-\lambda)(\lambda^2+4) \rightsquigarrow 1, \pm 2i.$$

1-eigenspace.

$$\lambda^2 = -4$$
$$\lambda = \sqrt{-4}$$
$$\lambda = \sqrt{4}\sqrt{-1}$$
$$= \pm 2i$$

2i-eigenvecs

$$\begin{pmatrix} 1-2i & 0 & 0 \\ 0 & -2i & -2 \\ 0 & 2 & -2i \end{pmatrix} \rightsquigarrow \begin{pmatrix} \boxed{1-2i} & 0 & 0 \\ 0 & \boxed{-2i} & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$-i \ast$ row 2

$(1-2i)x = 0$

$-2iy - 2z = 0 \to iy + z = 0$

$$\Rightarrow \begin{matrix} x = 0 \\ y = -z/i \\ z = z \end{matrix} \rightsquigarrow \begin{pmatrix} 0 \\ -1/i \\ 1 \end{pmatrix}$$

Complex eigenvalues

Find the complex eigenvalues and eigenvectors for

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{pmatrix}$$

Typical Exam Questions 5.5

- True/False. If v is an eigenvector for A with complex entries then $i \cdot v$ is also an eigenvector for A.
- True/False. If $(i, 1)$ is an eigenvector for A then $(i, -1)$ is also an eigenvector for A.
- If A is a 4×4 matrix with real entries, what are the possibilities for the number of non-real eigenvalues of A?
- Find the eigenvalues and eigenvectors for the following matrices.

$$\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \qquad \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

Spr 20

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

diag'able?

invertible?

$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ alg mult of 1 : 2

geom mult of 1 : 1

$(1-\lambda)^2$

$(1-\lambda)^2$

dim of 1-eigsp

diagable 1 has alg mult 2 & geom mult 2.

(xy-plane)

yes

invertible $\det = 1 \cdot 1 \cdot 3 \neq 0$.

5.5 WebWork.

- A real eigenval of a real matrix always has a real eigenvector. TRUE

$$\text{Null}(A - \lambda I)$$

- Every 3x3 matrix must a real eigenval.

TRUE ~~complex~~ non real ones come in pairs

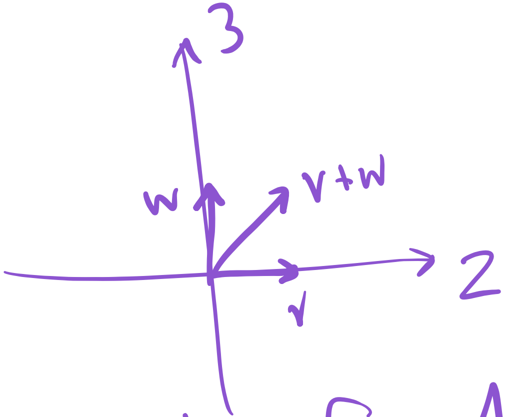
- A 5x5 real matrix has ~~even~~ odd. number of real eigenvals

Was false, but we fixed.

A $n \times n$ v, w eigenvectors for A

then $v + w$ is eigvec for A

False

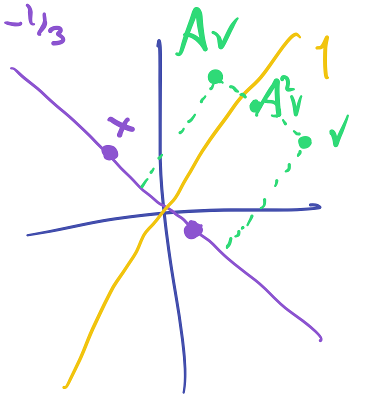


$$\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}2\\ 3\end{pmatrix}$$

A $n \times n$ v, w eigenvectors for A same eigenval

then $v + w$ is eigvec for A TRUE.

Spr 20 practice



$$A = CDC^{-1}$$

$$= \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -1/3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}^{-1}$$

F • Every nonzero vector is an eigenvector

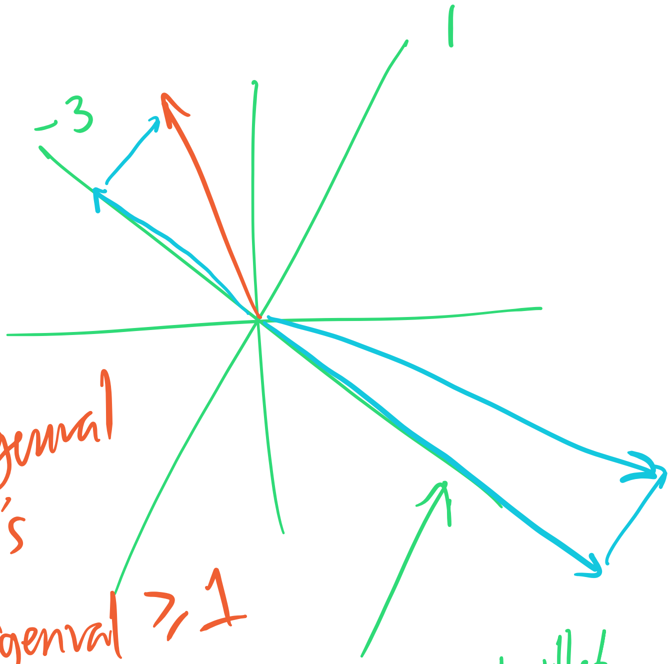
T • If $x = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $A^n x \longrightarrow 0$

F • Repeated mult by A pushes vectors to span $\cancel{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$ $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$A = CDC^{-1}$

$= \begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}\begin{pmatrix}-3 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}^{-1}$

biggest in abs val



General rule: Vectors get pulled towards eigensp w largest eigenval in abs val unless it's on another eigensp with eigenval ≥ 1