Section 2.2

Vector Equations and Spans

Outline of Section 2.2

- Learn the equivalences:

  $$\text{vector equations} \leftrightarrow \text{augmented matrices} \leftrightarrow \text{linear systems}$$

- Learn the definition of span
- Learn the relationship between spans and consistency

Linear Combinations

Is $\begin{pmatrix} 8 \\ 16 \\ 3 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$?

Write down an equation in order to solve this problem. This is called a vector equation.

Notice that the vector equation can be rewritten as a system of linear equations. Solve it!

Linear Combinations

Is $\begin{pmatrix} 8 \\ 16 \\ 3 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$?

Linear combinations, vector equations, and linear systems

In general, asking:

$$\text{Is } b \text{ a linear combination of } v_1, \ldots, v_k?$$

is the same as asking if the vector equation

$$x_1 v_1 + \cdots + x_k v_k = b$$

is consistent, which is the same as asking if the system of linear equations corresponding to the augmented matrix

$$\left(\begin{array}{cccc|c} | & | & & | & | \\ v_1 & v_2 & \cdots & v_k & b \\ | & | & & | & | \end{array}\right),$$

is consistent.

Compare with the previous slide! Make sure you are comfortable going back and forth between the specific case (last slide) and the general case (this slide).

The four ways

Four ways of saying the same thing:

- b is in Span$\{v_1, v_2, \ldots, v_k\}$ $\leftarrow$ geometry
- b is a linear combination of $v_1, \ldots, v_k$
- the vector equation $x_1 v_1 + \cdots + x_k v_k = b$ has a solution $\leftarrow$ algebra
- the system of linear equations corresponding to

$$\left(\begin{array}{cccc|c} | & | & & | & | \\ v_1 & v_2 & \cdots & v_k & b \\ | & | & & | & | \end{array}\right),$$

  is consistent.

▸ Demo ▸ Demo

Summary of Section 2.2

- vector equations $\leftrightarrow$ augmented matrices $\leftrightarrow$ linear systems
- Checking if a linear system is consistent is the same as asking if the column vector on the end of an augmented matrix is in the span of the other column vectors.

Typical exam questions

Is $\begin{pmatrix} 8 \\ 16 \\ 1 \end{pmatrix}$ in the span of $\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}$?

Write down the vector equation for the previous problem.

True/False: The vector equation $x_1 v_1 + \cdots + x_k v_k = 0$ is always consistent.