

Name Prof. M

## Mathematics 1553

Homework 1

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Section H / J

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1. A dietician is preparing a meal consisting of anchovies, bologna, and chili. Each ounce of anchovies contains 2 units of protein, 6 units of fat, and 4 units of carbohydrates. Each ounce of bologna contains 3 units of protein, 4 units of fat, and 1 unit of carbohydrates. Each ounce of chili contains 3 units of protein, 6 units of fat, and 2 units of carbohydrates. The meal must provide exactly 40 units of protein, 78 units of fat, and 34 units of carbohydrates.

Using the information in the problem, write down a system of linear equations that determines the number of ounces of each type of food that is needed for the meal.

$$\begin{aligned} 2a + 3b + 3c &= 40 \\ 6a + 4b + 6c &= 78 \\ 4a + b + 2c &= 34 \end{aligned}$$

Find the augmented matrix corresponding to the system of linear equations you found in the first part, compute its reduced row echelon form (showing all steps).

$$\left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 6 & 4 & 6 & 78 \\ 4 & 1 & 2 & 34 \end{array} \right) \longrightarrow \left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 0 & -5 & +3 & +42 \\ 0 & -5 & -4 & -46 \end{array} \right)$$

$$\longrightarrow \left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 0 & 5 & 3 & 42 \\ 0 & 0 & +1 & +4 \end{array} \right) \longrightarrow \left( \begin{array}{ccc|c} 2 & 3 & 0 & 28 \\ 0 & 5 & 0 & 30 \\ 0 & 0 & 1 & 4 \end{array} \right)$$

$$\longrightarrow \left( \begin{array}{ccc|c} 2 & 3 & 0 & 28 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right) \longrightarrow \left( \begin{array}{ccc|c} 1 & 3/2 & 0 & 14 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right)$$

$$\longrightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array} \right)$$

How many ounces of each type of food are needed for the meal?

$$\begin{aligned} a &= 5 \\ b &= 6 \\ c &= 4 \end{aligned}$$

If we instead say that the number of units of carbohydrates required is  $h$ , then for which  $h$  is there a solution to the system of linear equations? (Hint: it is not possible to eat a negative amount of bologna!)

$$\left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 6 & 4 & 6 & 78 \\ 4 & 1 & 2 & h \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 0 & +5 & +3 & +42 \\ 0 & -5 & -4 & h-80 \end{array} \right)$$

$$\rightarrow \left( \begin{array}{ccc|c} 2 & 3 & 3 & 40 \\ 0 & 5 & 3 & 42 \\ 0 & 0 & -1 & h-38 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 2 & 3 & 0 & 3h-74 \\ 0 & 5 & 0 & 3h-72 \\ 0 & 0 & 1 & 38-h \end{array} \right)$$

$$\rightarrow \left( \begin{array}{ccc|c} 2 & 3/2 & 0 & 3h/2-37 \\ 0 & 1 & 0 & 3h/5-72/5 \\ 0 & 0 & 1 & 38-h \end{array} \right)$$

$$\rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & 3h/2-37-3/2(3h/5-72/5) \\ 0 & 1 & 0 & 3h/5-72/5 \\ 0 & 0 & 1 & 38-h \end{array} \right)$$

$$= \left( \begin{array}{ccc|c} 1 & 0 & 0 & (3h-77)/5 \\ 0 & 1 & 0 & (3h-72)/5 \\ 0 & 0 & 1 & 38-h \end{array} \right)$$

For all  $z$  to be nonnegative,  
 need  $h \geq 77/3$  (redundant)  
 $h \geq 72/3 = 24$  (redundant)  
 $h \leq 38$

$$\rightsquigarrow 24 \leq h \leq 38.$$