Name ________________________

Mathematics 1553

Written Homework 2
Prof. Margalit
Section: H / J (circle one)
Subsection: left / center / right (circle one)
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1. Let A be a 3×2 matrix and let

$$v = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \text{and} \quad c = \begin{pmatrix} 10 \\ -5 \\ 15 \end{pmatrix}.$$

If Av is equal to c, is it true that the matrix equation $Ax = b$ is consistent? Answer *yes/no/maybe* and explain your answer.

$A\vec{v} = \vec{c}$: given to be consistent

$\Rightarrow A\vec{v} = k\vec{b}$ $\because$ $5\vec{b} = \vec{c}$

$\Rightarrow A\vec{x} = \vec{b}$ for $\frac{1}{k}\vec{v} = \vec{x}$

$\hookrightarrow$ $A\vec{x} = \vec{b}$ is consistent

$\boxed{\text{YES}}$

2. Suppose we have a collection of objects in $\mathbb{R}^n$ located at the points $v_1, \ldots, v_k$ and having masses $m_1, \ldots, m_k$. The *center of mass* of the collection of objects is:

$$\frac{m_1 v_1 + \cdots + m_k v_k}{m_1 + \cdots + m_k}$$

Find the center of mass of the collection of objects that all weigh 1 gram and are located at the points $(0, 1)$, $(8, 1)$, and $(2, 4)$ in $\mathbb{R}^2$.

$$\frac{1}{3}\begin{bmatrix} 0 & 8 & 2 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$= \left(\frac{10}{3}, 2\right)$$

Determine how to distribute an additional mass of 6 grams at the three points $(0, 1)$, $(8, 1)$, and $(2, 4)$ so that the center of mass moves to $(2, 2)$. *Hint: Add masses w_1, w_2, w_3 to the three points so that $w_1 + w_2 + w_3 = 6$.*

$$\frac{1}{9}\begin{bmatrix} 0 & 8 & 2 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1+w_1 \\ 1+w_2 \\ 1+w_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

But, $w_1 + w_2 + w_3 = 6$

$\Rightarrow w_3 = 6 - w_1 - w_2$

$$\frac{1}{9}\begin{bmatrix} 10 + 8w_2 + 2w_3 \\ 6 + w_1 + w_2 + 4w_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Equations:

$$w_1 + w_2 + w_3 = 6$$
$$8w_2 + 2w_3 = 8$$
$$w_1 + w_2 + 4w_3 = 12$$

Solution: $w_1 = 3.5g \ (4.5)$

$w_2 = 0.5g \ (1.5)$

$w_3 = 2\,g \ (3)$