1. Let $A$ be a $3 \times 2$ matrix and let

$$v = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \text{and} \quad c = \begin{pmatrix} 10 \\ -5 \\ 15 \end{pmatrix}.$$ 

If $Av$ is equal to $c$, is it true that the matrix equation $Ax = b$ is consistent? Answer yes/no/maybe and explain your answer.

If $Av = c$, then $Ax = b$ is consistent.

$A\vec{v} = \vec{c}$, given to be consistent

$\Rightarrow A\vec{v} = k\vec{b}$, hence $b = \vec{c}$

$\Rightarrow A\vec{x} = \vec{b}$ for $\frac{1}{k} \vec{v} = \vec{x}$

$\Rightarrow A\vec{x} = \vec{b}$ is consistent

2. Suppose we have a collection of objects in $\mathbb{R}^n$ located at the points $v_1, \ldots, v_k$ and having masses $m_1, \ldots, m_k$. The center of mass of the collection of objects is:

$$\frac{m_1v_1 + \cdots + m_kv_k}{m_1 + \cdots + m_k}$$

Find the center of mass of the collection of objects that all weigh 1 gram and are located at the points $(0, 1)$, $(8, 1)$, and $(2, 4)$ in $\mathbb{R}^2$.

$$\frac{1}{3} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

$$= \left( \frac{10}{3}, 2 \right)$$
Determine how to distribute an additional mass of 6 grams at the three points (0, 1), (8, 1), and (2, 4) so that the center of mass moves to (2, 2). Hint: Add masses $w_1$, $w_2$, $w_3$ to the three points so that $w_1 + w_2 + w_3 = 6$.

$$\frac{1}{9} \begin{bmatrix} 0 & 8 & 2 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 + w_1 \\ 1 + w_2 \\ 1 + w_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

But $w_1 + w_2 + w_3 = 6$ implies $w_3 = 6 - w_1 - w_2$

$$\frac{1}{9} \begin{bmatrix} 10 + 8w_2 + 2w_3 \\ 6 + w_1 + w_2 + 4w_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

**Equations:**

1. $w_1 + w_2 + w_3 = 6$
2. $8w_2 + 2w_3 = 8$
3. $w_1 + w_2 + 4w_3 = 12$

**Solution:**

$w_1 = 3.5 \text{ g (4.5)}$

$w_2 = 0.5 \text{ g (1.5)}$

$w_3 = 2 \text{ g (3)}$