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Name Prof. M

Section H J

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### Mathematics 1553

Written Homework 3

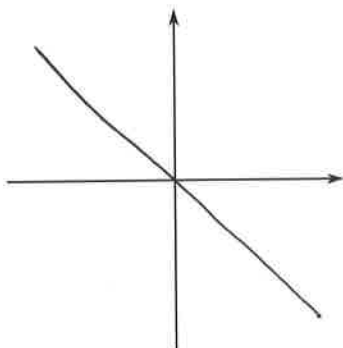
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1. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

Find all  $b$  so that  $Ax = b$  is consistent. Plot the set of such  $b$ . Explain your answer.

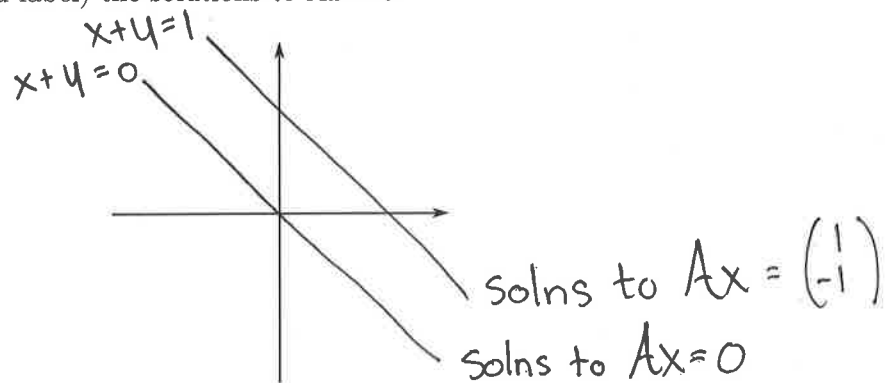


$Ax=b$  is consistent when  $b$  is in the span of the cols of  $A$ :

$$\text{span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

This is the line  $y = -x$ .

Plot (and label) the set of solutions to  $Ax = 0$ . Choose a nonzero  $b$  so that  $Ax = b$  is consistent and plot (and label) the solutions to  $Ax = b$



Is there a vector  $b$  in  $\mathbb{R}^2$  so that the solutions to  $Ax = b$  is the line  $x + y = 6$ ? If so, find such a  $b$ ; if not, explain why not.

$x + y = 6$  is parallel to  $x + y = 0$  so there is a chance.

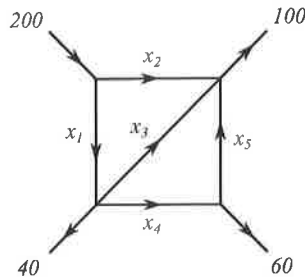
$$\left( \begin{array}{cc|c} 1 & 1 & b_1 \\ -1 & -1 & -b_1 \end{array} \right) \rightsquigarrow \left( \begin{array}{cc|c} 1 & 1 & b_1 \\ 0 & 0 & 0 \end{array} \right)$$

Solns:  $x + y = b_1$

So  $b_1 = 6$

$$b = \begin{pmatrix} b_1 \\ -b_1 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$$

2. The traffic in the town square is described by the following diagram (The arrows indicate the directions of one way streets and the labels indicate the number of cars per hour).



Write down a system of linear equations that describes the flow of traffic in the town square.

$$\begin{aligned}x_1 + x_2 &= 200 \\x_2 + x_3 + x_5 &= 100 \\x_4 - x_5 &= 60 \\x_1 - x_3 - x_4 &= 40\end{aligned}$$

Write down the corresponding augmented matrix and find its reduced row echelon form.

$$\left( \begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 & -1 & 60 \\ 1 & 0 & -1 & -1 & 0 & 40 \end{array} \right)$$

$$\rightsquigarrow \left( \begin{array}{ccccc|c} 1 & 0 & -1 & 0 & -1 & 100 \\ 0 & 1 & 1 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

What is the parametric form of the solution to the linear system?

$$\begin{pmatrix} 100 \\ 100 \\ 0 \\ 60 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

What will be the traffic on each street if the street labeled  $x_4$  is closed for construction?

To make  $x_4 = 0$  set  $x_5 = -60$

Need to change the direction of  
the  $x_5$  street.

$$\rightarrow \begin{pmatrix} x_3 + 40 \\ -x_3 + 160 \\ x_3 \\ 0 \\ -60 \end{pmatrix}$$

What if instead the street labeled  $x_1$  is closed for construction? How can traffic be rerouted?  
(You may need to change the directions of some one way streets.)

Set  $x_3 = -100$  and again reverse  
the direction so this  
becomes  $+100$ .