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Section H J
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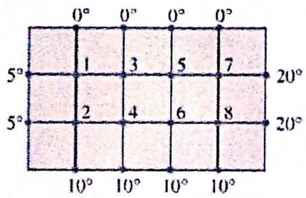
Row number 1 2 3 4 5 6 7 8

Mathematics 1553

Written Homework 5
Prof. Margalit
26 February 2016

1. Say we want to find the temperatures at the interior points in this grid:



As in problem 7 on WebWork assignment 1.1, we do this by solving $Ax = b$ where

$$A = \begin{pmatrix} 4 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 4 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 4 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 4 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 4 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 \end{pmatrix}$$

and $b = (5, 15, 0, 10, 0, 10, 20, 30)$. We say that A is a band matrix because all of the nonzero entries are near the diagonal. Find an LU decomposition of A.

Use Matlab:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.25 & -0.0667 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2667 & -0.2857 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.2679 & -0.0833 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2917 & -0.2921 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.2697 & -0.0861 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.2948 & -0.2931 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.75 & -0.25 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.733 & -1.0667 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.4286 & -0.2857 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.7083 & -1.0833 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.3919 & -0.2912 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.7052 & -1.0861 \\ 0 & -0 & 0 & 0 & 0 & 0 & 0 & 3.3868 \end{bmatrix}$$

Use your LU factorization to solve $Ax = b$.

Solve $L\vec{y} = \vec{b}$; $U\vec{x} = \vec{y}$

since U has pivot in every column.

$$\vec{y} = L^{-1}\vec{b} = \begin{pmatrix} 5 \\ 16.25 \\ 2.33 \\ 15 \\ 1.875 \\ 14.923 \\ 21.791 \\ 40.7873 \end{pmatrix}$$

$$\vec{x} = U^{-1} y = \begin{pmatrix} 3.957 \\ 6.589 \\ 4.239 \\ 7.397 \\ 5.603 \\ 8.761 \\ 9.412 \\ 12.043 \end{pmatrix}$$

Use a computer program to find A^{-1}, for instance: `http://www.bluebit.gr/matrix-calculator/`

$$A^{-1} = \begin{pmatrix} 0.2953 & 0.0866 & 0.0945 & 0.0509 & 0.0318 & 0.0227 & 0.01 & 0.0082 \\ 0.0866 & 0.2953 & 0.0509 & 0.0945 & 0.0227 & 0.0318 & 0.0082 & 0.01 \\ 0.0945 & 0.0509 & 0.3271 & 0.1093 & 0.1045 & 0.0591 & 0.0318 & 0.0227 \\ 0.0509 & 0.0945 & 0.1093 & 0.3271 & 0.0591 & 0.1045 & 0.0227 & 0.0318 \\ 0.0318 & 0.0227 & 0.1045 & 0.0591 & 0.3271 & 0.1093 & 0.0945 & 0.0509 \\ 0.0227 & 0.0318 & 0.0591 & 0.1045 & 0.1093 & 0.3271 & 0.0509 & 0.0945 \\ 0.01 & 0.0082 & 0.0318 & 0.0227 & 0.0945 & 0.0509 & 0.2953 & 0.0866 \\ 0.0082 & 0.01 & 0.0227 & 0.0318 & 0.0509 & 0.0945 & 0.0866 & 0.2953 \end{pmatrix}$$

If A was a 100×100 band matrix, which do you think would take less computer memory to store, A^{-1} or L and U?

For the LU decomposition of the band matrix A, L and U both have $(100+99+98) = 297$ nonzero entries, in total of 594, while A^{-1} has 10^4 nonzero entries. Therefore, L and U would take less memory.