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Name Sol.

Section H J
Subsection left center right

# Mathematics 1553
Written Homework 9
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15 April 2016

1. Consider the matrix

$$A = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{pmatrix}$$

Compute $A^T A$.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What does your answer say about the columns of $A$?

They are Orthogonal/orthonormal because each column dotted with the other columns equals 0, while each column dotted with itself equals one.

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Choose two linearly independent vectors $u$ and $v$ in $\mathbb{R}^4$ (choose them so no entry is equal to 0). Write them here.

$u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

$v = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$

Compute the following.

$\|u\| = \sqrt{1^2 + 1^2 + 1^2 + 2^2} = \sqrt{7}$

$\|v\| = \sqrt{1^2 + 1^2 + 3^2 + 1^2} = \sqrt{12}$

$u \cdot v = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 3 + 2 \cdot 1 = 7$

Now compute the following.

$T_A(u) = A \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$

$T_A(v) = A \cdot \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

$\|T_A(u)\| = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{7}$

$\|T_A(v)\| = \sqrt{3^2 + (-1)^2 + 1^2 + (-1)^2} = \sqrt{12}$

$T_A(u) \cdot T_A(v) = 7$

Summarize what the these calculations suggest about $A$ (or rather $T_A$).

→ $T_A$ doesn't change vector length

→ $T_A$ doesn't change angle between two vectors

*Extra credit (two points).* Prove your hypothesis about $A$.

$T_A(u) \cdot T_A(v) = (Au) \cdot (Av) = (Au)(Av)^T = AA^T(uv^T) = uv^T$

Thus, $AA^T = I_n$