Announcements April 25

- CIOS open: additional dropped quiz for 85% response rate (measured at start of final)
- Review on Friday April 29 at 6:30-8:00 in Skiles 154
- Final Exam Wed May 4 8:00-10:50 (Sec H) and Mon May 2 2:50-5:40 (Sec J)
- Office Hours Tue 2-3 and Wed 2-3
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Shivang Tue 5-6, Baishen Wed 4-5, Matt Thu 3-4
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Inconsistent

What is the difference between a system of equations being inconsistent and a system having infinitely many solutions?

$Ax = b$

Midterm 2

inconsistent: no solution

How to check

inconsistent: pivot in last col of $(A|b)$

one soln: pivot in each col of A

∞ many soln: everything else.

One-to-one and onto

Do the following give linear transformations that are one-to-one? onto?

$$\begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 9 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$A = m \times n$

T_A is a function $\mathbb{R}^n$ to $\mathbb{R}^m$

input: v

output: Av

m, n, $n \times 1$

one-to-one: • $v \neq w$ then $T_A(v) \neq T_A(w)$

• if pivot in each col

onto: all outputs occur.

• if pivot in each row

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

not one-to-one: $T_A\begin{pmatrix}0\\0\\1\end{pmatrix} = T_A\begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

or onto: $\begin{pmatrix}0\\0\\1\end{pmatrix}$ not linear combo of the cols.

Exam 1

Problem 5

(a) Consider the matrix

$$A = \begin{pmatrix} \boxed{1} & -2 \\ 0 & \boxed{2} \\ 0 & 0 \end{pmatrix}$$

and let T_A be the associated linear transformation.

Is T_A one-to-one? yes, pivot in each col.

Find one nonzero vector b in the range of T_A.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

or any non-0 lin combo of the cols.

LU Decompositions

Consider the matrix

$$A = \begin{pmatrix} -5 & 2 \\ 5 & 2 \\ 15 & 18 \end{pmatrix}$$

Find an LU decomposition of A.

$A = m \times n$
$U = m \times n$
$L = m \times m$

$U =$ row red. of A
only using "lower" row replacement
in order (col by col)

$L =$ keeps track; unit lower Δ.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & +6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 2 \\ 5 & 2 \\ 15 & 18 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -5 & 2 \\ 0 & 4 \\ 0 & 24 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -5 & 2 \\ 0 & 4 \\ 0 & 0 \end{pmatrix} = U$$

LU Decompositions

Use your LU decomposition from above to solve the equation $Ax = b$ where

$$b = \begin{pmatrix} 7 \\ -3 \\ 3 \end{pmatrix} \qquad A = LU$$

Show clearly the two steps.

Step 1. $Ly = b$

Step 2. $Ux = y$.

Exam 1

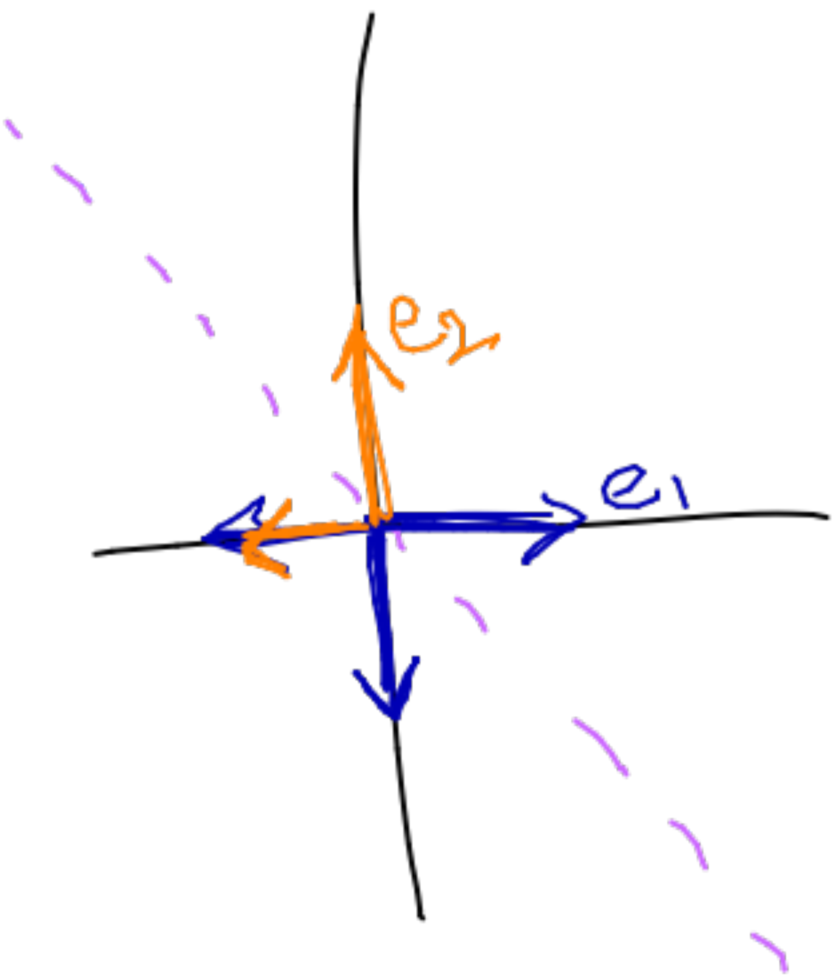
Problem 5

(b) Find a matrix A so that T_A is the linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ obtained by first reflecting about the line $y = -x$ and then rotating clockwise by $\pi/2$. (Note: this problem is completely independent of the first problem on this page—the two As have nothing to do with each other.)

To find the matrix from a linear transformation T:

$$\begin{pmatrix} T(e_1) & \cdots & T(e_n) \end{pmatrix}$$

cols.



$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Least Squares

Consider the points $(0,0)$, $(0,1)$, $(1,-1)$, and $(-1,0)$ in the xy–plane.

Find the best fit line.

Find the best fit quadratic $y = f(x)$.

Find the best fit cubic $y = f(x)$.

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