Announcements April 6

- WebWork 5.3 and 5.5 due Thursday
- Supplemental WebWork 5.5 does not count for your grade
- Homework 7 due Friday April 8
- Midterm 3 in class Friday April 8 on Chapter 5
- Office Hours Wed 2-3 and Thu 2-3
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Shivang Tue 5-6, Baishen Wed 4-5, Matt Thu 3-4
- Math Lab, Clough 280
  - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
  - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
  - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Midterm 3

Piazza Questions

Finding Eigenvectors and Eigenvalues

Linear transformations

Suppose that A is a 2×2 matrix and the associated linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ is orthogonal projection onto the y-axis. List the eigenvalues of A (if there are any) and give a basis for each corresponding eigenspace.

Finding Eigenvectors and Eigenvalues

Linear transformations

Suppose that A is a 2×2 matrix and that the associated linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ is rotation about the origin by $\pi/4$. List the eigenvalues of A (if there are any) and give a basis for each corresponding eigenspace.

Finding Eigenvectors and eigenvalues

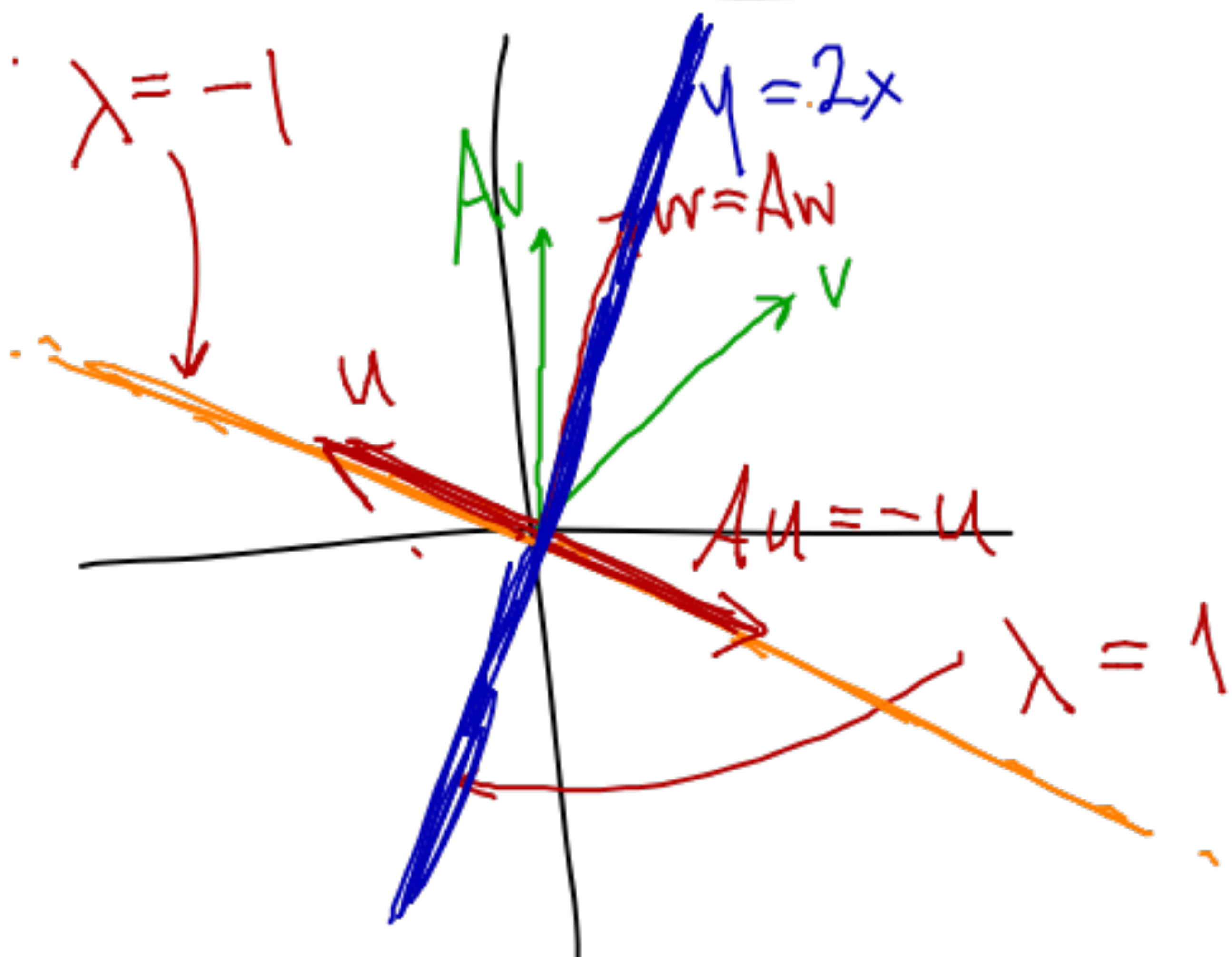
Linear transformations

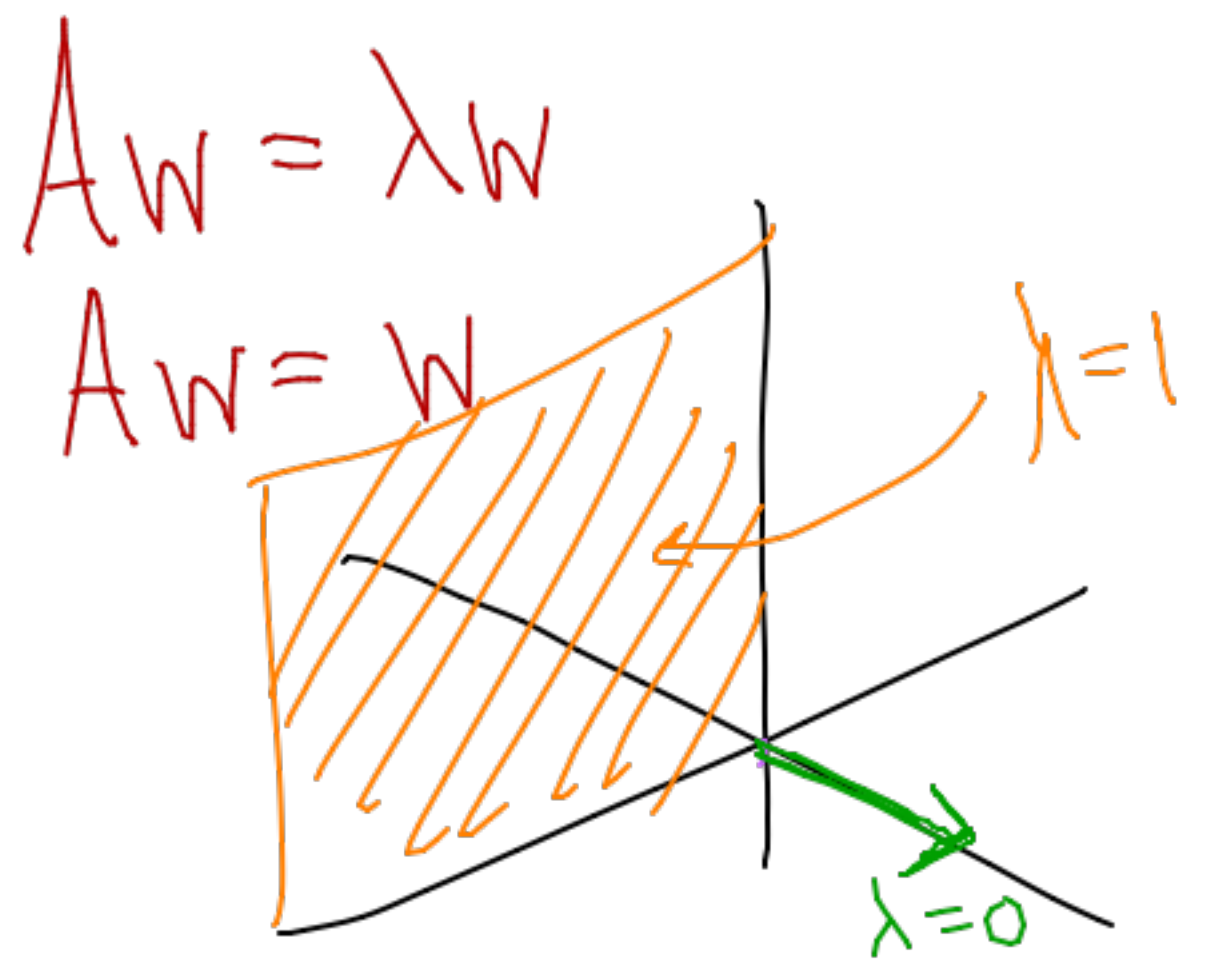
Find the eigenvectors/eigenvalues for A without doing any matrix calculations.

- T_A = identity transformation of $\mathbb{R}^3$
- T_A = orthogonal projection to xz-plane in $\mathbb{R}^3$
- T_A = counterclockwise rotation by $\pi/4$ in $\mathbb{R}^2$
- T_A = reflection about $y = 2x$

Av=v all v

λ=1 1-eigenspace is $\mathbb{R}^3$.





Diagonalization

To find λ-eigenspace solve $(A-\lambda I)v = 0$.

Diagonalize, if possible.

$$A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{pmatrix}$$

Char poly is ~~$(\lambda-5)^2(\lambda-1)$~~ or $(\lambda-5)(\lambda-1)^2$

The eigenvalues of A are 5 and 1.

$\boxed{\lambda=1}$ $\begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

dim 1-eigenspace = 2 ≤ alg mult. of 1

$\rightarrow x_2\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ← basis for 1-eigenspace

$\boxed{\lambda=5}$ $\begin{pmatrix} -3 & 2 & -1 \\ 1 & -2 & -1 \\ -1 & -2 & -3 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -2 & -1 \\ -1 & -2 & -3 \\ -3 & 2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & -1 \\ 0 & -4 & -4 \\ 0 & -4 & -4 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightsquigarrow x_3\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

Diagonalization

$$A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix}\begin{pmatrix} \text{same as first.} \end{pmatrix}^{-1}$$

Complex eigenvalues

Find eigenvalues and eigenvectors:

$$(A - \lambda I)x = 0$$

$$A = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}$$

eigenvalues: $\det \begin{pmatrix} 5-\lambda & -5 \\ 1 & 1-\lambda \end{pmatrix} = (5-\lambda)(1-\lambda) + 5$

$$= \lambda^2 - 6\lambda + 10$$

$$\lambda = \frac{6 \pm \sqrt{36-40}}{2} = \frac{6 \pm \sqrt{-4}}{2} = 3 \pm i$$

eigenvectors:

$\lambda = 3+i$: $\begin{pmatrix} 5-(3+i) & -5 \\ 1 & 1-(3+i) \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2-i & -5 \\ 0 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 5 \\ 2-i \end{pmatrix}$ e-vector.

$\lambda = 3-i$: $\overline{\begin{pmatrix} 5 \\ 2-i \end{pmatrix}} = \begin{pmatrix} 5 \\ 2+i \end{pmatrix}$

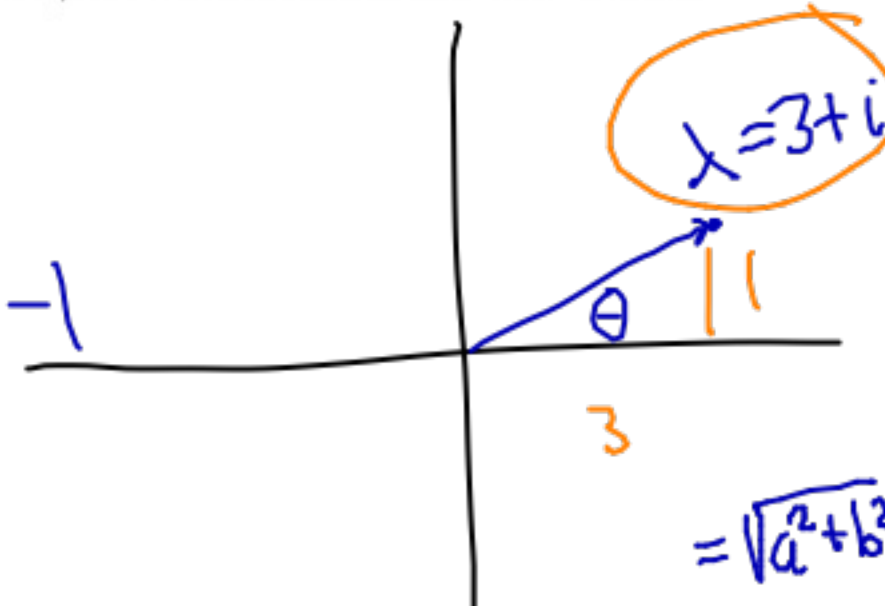
Complex eigenvalues

Find a rotation-plus-scaling matrix B that A is similar to, and find a C so that $A = CBC^{-1}$.

$$A = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}$$

$= 3 + 1 \cdot i$

$\lambda = 3 + i = a - bi \quad a = 3,\ b = -1$

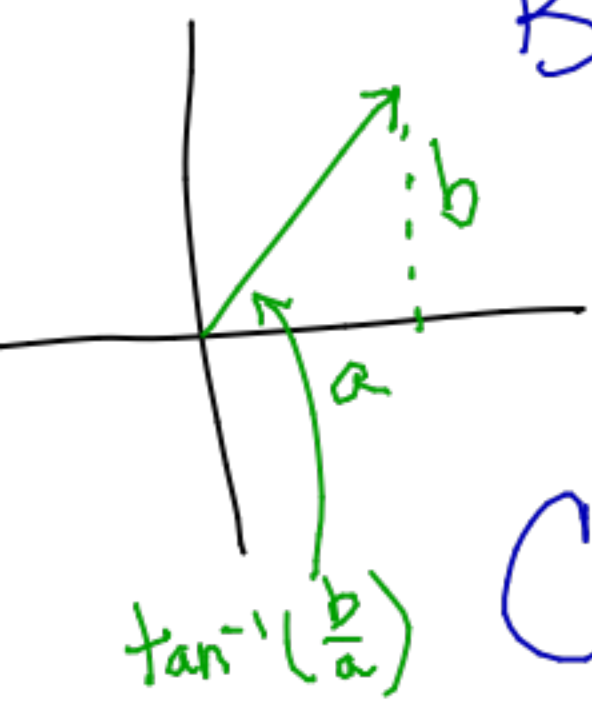


$v = \begin{pmatrix} 5 \\ 2-i \end{pmatrix} = \begin{pmatrix} 5 + 0i \\ 2 + (-1)i \end{pmatrix}$

scales by: $|\lambda| = \sqrt{a^2+b^2} = \sqrt{3^2+1^2} = \sqrt{10}$

$B = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} = \sqrt{10}\begin{pmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ -1/\sqrt{10} & 3/\sqrt{10} \end{pmatrix}$

rotates by: $\theta = \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{-b}{a}\right)$



$C = \left(\text{Re } v \mid \text{Im } v \right) = \begin{pmatrix} 5 & 0 \\ 2 & -1 \end{pmatrix}$

Markov matrices

Suppose that A is a matrix where the entries in each column add up to 1 (for example, the matrices in the Google Pagerank algorithm). Show that A has an eigenvalue equal to 1. *Hint: find an eigenvector for* A^T*.*

Markov matrices

Rental cars

A rental car agency has two locations. One fourth of the cars from the first location get returned to the first location and three-fourths to the second. Two-thirds of the cars from the second location get returned to the first location and one-third to the second. How should the agency distribute their cars in order to minimize the number of cars that need to be shuttled?

Similar matrices

Explain why similar matrices have the same eigenvalues.