Announcements Feb 1

- Extra credit for WebWork bugs: post a screen shot and a clear explanation
- WebWork 1.5 due Friday
- Written Homework 3 due Friday
- Quiz 3 on Friday on Section 1.5
- Midterm 1 in class Friday Feb 12
- My Office Hours Tuesday and Wednesday 2-3, after class, and by appointment
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 1.7

Linear Independence

Linear Independence

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

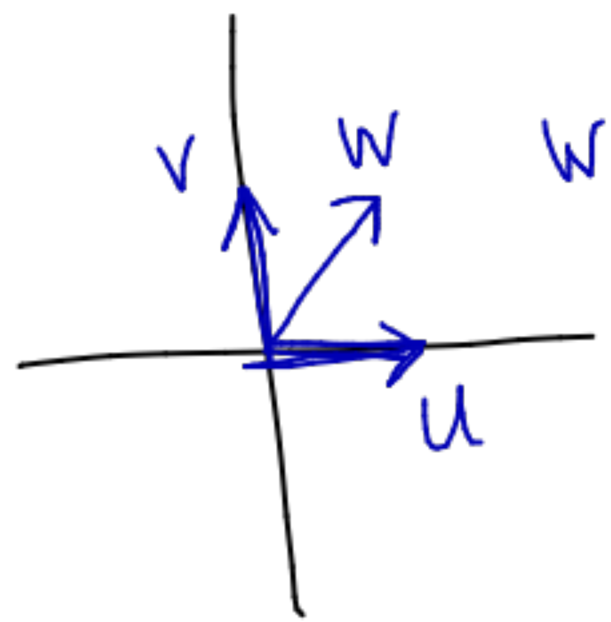
has only the trivial/zero solution. It is linearly dependent otherwise.

real numbers

So, linearly dependent means there are $c_1, c_2, \ldots, c_k$ not all 0 so that

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

This is the *linear dependence* relation.



$w - v - u = 0$

Linear Independence

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

has only the trivial solution.

Fact. The cols of A are linearly independent $\Leftrightarrow Ax = 0 \ldots$ has only 0 soln
$\Leftrightarrow A \ldots$ pivot in each col.

Why?

Linear Independence

Example. Is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

$$2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

or row red

$\rightsquigarrow$ row of 0's.

Discussion Question

Question

For what values of h are the vectors linearly ~~in~~dependent?

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ h \end{pmatrix}, \begin{pmatrix} 1 \\ h \\ 1 \end{pmatrix}, \begin{pmatrix} h \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 1 & h \\ 1 & h & 1 \\ h & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & h \\ 0 & h-1 & 1-h \\ 0 & 1-h & 1-h^2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 1 & h \\ 0 & h-1 & 1-h \\ 0 & 0 & 2-h-h^2 \end{pmatrix} \to h = -2, \; = 1$$

Linear Independence

One vector

When is $\{v\}$ is linearly dependent?

$$v = 0$$

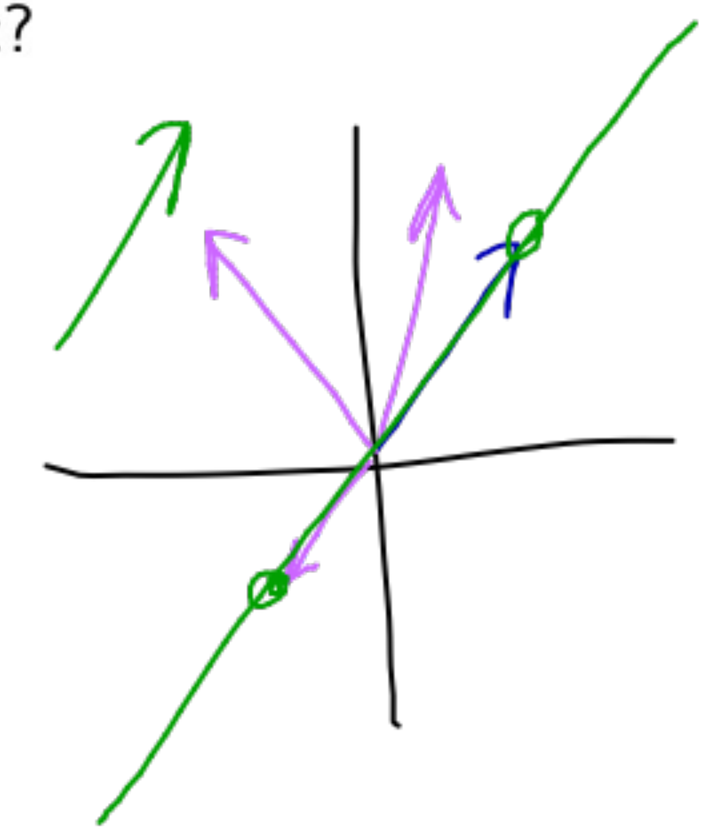
$$1 \cdot v = 0$$

Linear Independence

Two vectors

When is $\{v_1, v_2\}$ is linearly dependent?

- on same line thru 0
- one is a scalar mult of the other.



Linear Independence

Any number of vectors

When is the set $\{v_1, v_2, \dots, v_k\}$ linearly in dependent?

$v_1 \neq 0$

v_2 not on v_1-line $= \text{span}\{v_1\}$

v_3 not in $\text{span}\{v_1, v_2\} = \text{plane}$.

etc.

Span and Linear Independence

Example. Is $\left\{ \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \right\}$ linearly independent?

$$\text{span}\left\{ \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 7 \\ 0 \end{pmatrix} \right\} = xy\text{-plane in } \mathbb{R}^3$$

$\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ not in xy-plane ✓

Span and Linear Independence

Two More Facts

Fact 1. Say $v_1, \ldots, v_k$ are in $\mathbb{R}^n$. If $k > n$, then $\{v_1, \ldots, v_k\}$ is

dep

Fact 2. If one of $v_1, \ldots, v_k$ is 0, then $\{v_1, \ldots, v_k\}$ is

$v_1 = 0$

dep $\Longrightarrow 5 \cdot v_1 = 0$