Announcements Feb 10

Review
6:30 today
Skiles 249

- Please complete mid-semester CIOS evaluations this week
- WebWork 1.7 and 1.8 due Thursday
- WebWork 1.9 extra credit, due Thursday
- Midterm 1 in class this week Friday Feb 12 on Chapter 1
- Office Hours Tuesday and Wednesday 2-3, after class, and by appointment in Skiles 244 or 236
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Chapter 1

Review

Linear systems

We want to solve linear systems. Why? Engineering, econ, chem, physics, ...
$\rightsquigarrow$ matrices and row echelon form.

$Ax = b$ is consistent $\Leftrightarrow$ $(A \mid b)$ has... no pivot in last col
$\Leftrightarrow$ b is... in span of cols of A.

So checking if w is in the span of $\{v_1, \ldots, v_k\}$ is the same as checking if...

$(v_1 \cdots v_k \mid w)$ has no pivot in last col.

Also: $Ax = b$ is consistent for all b $\Leftrightarrow$ A has pivot in each row

$\Longleftrightarrow$ cols of A span $\mathbb{R}^m$

$A = m \times n$ matrix

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{c|c|c} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{array}\right)$$

Parametric solutions

Solutions to $Ax = 0$ can be written in parametric form.

$$x_2\begin{pmatrix}1\\1\\0\end{pmatrix} + x_3\begin{pmatrix}-5\\0\\1\end{pmatrix}$$

$\#$ free vars = #cols − # pivots

$\rightsquigarrow$ can write solution as a... Span

so the solutions give a... pt, line, plane, ... thru origin.

Solutions to $Ax = b$ is a... translate of solns to $Ax=0$.

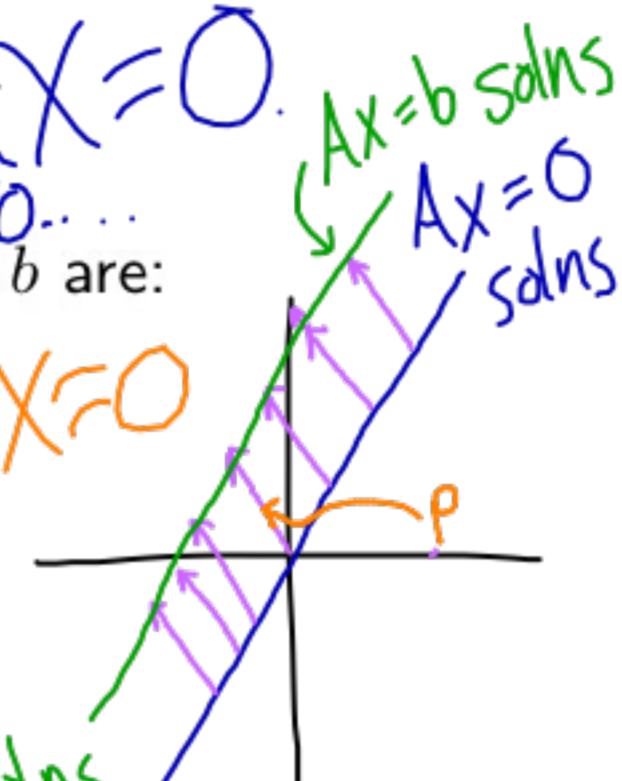
either nothing or: another pt/line/plane parallel to....

If p is one single solution to $Ax = b$, then the solutions to $Ax = b$ are:

$p + v$ v is any soln to $Ax=0$

example: $\begin{pmatrix}1\\1\\1\end{pmatrix} + x_2\begin{pmatrix}1\\1\\0\end{pmatrix} + x_3\begin{pmatrix}-5\\0\\1\end{pmatrix}$

$\rightsquigarrow$ parametric form for solutions to $Ax = b$.



Linear independence

Q. How many vectors do you need in the parametric form of the solution?

$\rightsquigarrow$ linear independence

$\{v_1, \dots, v_k\}$ is linearly independent $\Leftrightarrow$ $\underset{\displaystyle A}{\overset{\shortparallel}{(v_1 \cdots v_k)}}$ has pivot in each col.

$$\boxed{\begin{gathered} x_1 v_1 + \cdots + x_k v_k = 0 \\ Ax = 0 \end{gathered}}$$

or $v_1 \neq 0$

v_2 not in span of v_1 (not a multiple)

v_3 not in span of v_1, v_2

etc...

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{pmatrix}$$

Linear transformations

$m \left(\quad {}_n \quad \right) \left(\right) = \left(\right)$

$n\times 1 \qquad m \times 1$

A $= m \times n$ matrix $\rightsquigarrow$ matrix transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ $\quad T_A(v) = Av$

Every linear transformation is a matrix transformation, that is, if $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation then...

Linear

$T(u+v) = T(u) + T(v)$

$T(cv) = cT(v)$

$$A = \begin{pmatrix} T(e_1) & \cdots & T(e_n) \end{pmatrix}$$

Range of T_A = span of cols of A.

$T_A(v) = Av$ = lin comb of cols of A.

T_A is onto $\Leftrightarrow$ span of cols of A in $\mathbb{R}^m$

$\Leftrightarrow$ pivot in each row

$\Leftrightarrow$ $Ax = b$ consistent for all b in $\mathbb{R}^m$

different inputs → diff outputs

T_A is one-to-one $\Leftrightarrow$ $Ax = 0$ only 0 sln

$\Leftrightarrow$ A has pivot in each col

$\Leftrightarrow$ cols of A lin ind.

Linear independence

How can you tell if a set of vectors in linearly indepdent without row reduction?

Poll

Which of the following are linearly independent?

1. $(3,3,4),\ (0,10,20),\ (0,-1,-2)$
2. $(3,3,4),\ (0,5,7),\ (0,6,8)$
3. $(3,3,4),\ (0,1,0),\ (0,0,\sqrt{2})$
4. $(5,7,0),\ (6,8,0),\ (3,3,4)$

Quiz 3

Homogeneous vs. nonhomogeneous

1. Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$$

Write in parametric form the solutions to $Ax = b$.

$$x_1 = -2 - x_3$$
$$x_2 = 8 - x_3$$
$$x_3 = \cancel{\text{free}}\ x_3$$

$$\begin{pmatrix} -2 \\ 8 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

line

Write in parametric form the solutions to $Ax = 0$ (same A as above).

$$x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

True/False. There is a vector b in $\mathbb{R}^2$ so that the set of solutions to $Ax = b$ is the yz–plane in $\mathbb{R}^3$ (same A as above). Explain your answer.

No: yz-plane is not a line

Linear independence with parameters

Find the value(s) of h for which the vectors are linearly dependent?

$$(1, 5, -3), \quad (-2, -9, 6), \quad (3, h, -9)$$

$$\begin{pmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 3 \\ 0 & -19 & h-15 \\ 0 & 0 & \cancel{0}\; h-2 \end{pmatrix}$$

all h.

Practice exam

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3×4

$T_A : \mathbb{R}^4 \to \mathbb{R}^3$

and let T_A be the associated linear transformation.

What is the domain of T_A? $\mathbb{R}^4$

Is T_A one-to-one? No: col w/o pivot.

Is T_A onto? No: row w/o pivot

What is the dimension of the set of solutions to $Ax = 0$ (that is, how many free variables)? two

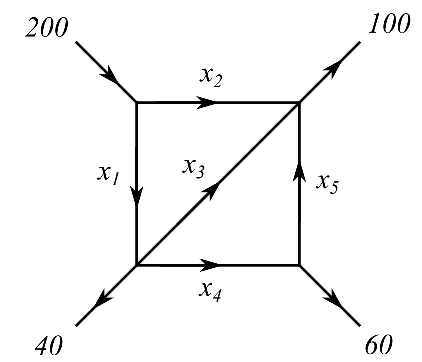
Does $Ax = b$ have a solution for every b in $\mathbb{R}^3$? No: T_A not onto

What is the span of the columns of A?

xy-plane

Written homework # 3

2. The traffic in the town square is described by the following diagram:



Write down a system of linear equations that describes the flow of traffic.

Write down the augmented matrix and find its reduced row echelon form.

What is the parametric form of the solution?

What will be the traffic on each street if the street x_4 is closed?

What if instead the street labeled x_1 is closed for construction?