Chapter 2

Matrix Algebra

Section 2.1

Matrix Operations

Terminology

Suppose A is an $m \times n$ matrix.

a_{ij} or A_{ij} is the ijth entry

$$A = \begin{pmatrix} 6 & -7 \\ 2 & 3 \end{pmatrix}$$

$A_{2,1}$ = entry in row 2, col 1 = 2

$A_{1,2} = -7$

a_{ii} are diagonal entries

diagonal matrix: all square & non-diagonal entries are 0 $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

identity matrix: diagonal matrix with 1's on the diagonal $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

zero matrix: all entries are 0 $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

the transpose of A is denoted A^T and has ij entry a_{ji}

$$\begin{pmatrix} 7 & 1 \\ 5 & 2 \end{pmatrix}^T = \begin{pmatrix} 7 & 5 \\ 1 & 2 \end{pmatrix}$$

Sums and Scalar Multiples

Same as for vectors: component-wise, so matrices must be same size to add.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

$A + B = B + A$

$(A + B) + C = A + (B + C)$

$r(A + B) = rA + rB$

$(r + s)A = rA + sA$

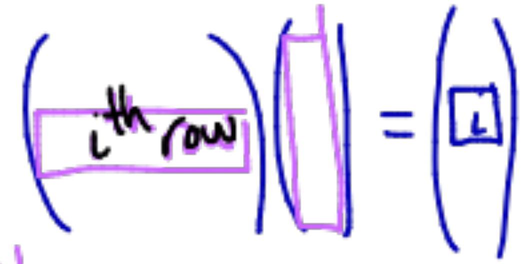
$(rs)A = r(sA)$

$A + 0 = A$

$r, s = \#s$

Matrix Multiplication

If A is $m \times n$ and B is $n \times p$, then AB is $m \times p$ and



$$(AB)_{ij} = \text{ith row of A times jth col of B}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1\cdot5+2\cdot7 & 1\cdot6+2\cdot8 \\ 3\cdot5+4\cdot7 & 3\cdot6+4\cdot8 \end{pmatrix}$$
$$= \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

Or: jth col of AB is ... A times jth col of B

Discussion Question

Are there nonzero matrices A and B with $AB = 0$?

1. Yes
2. No

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Matrix Multiplication and Linear Transformations

Fact. $T_{AB} = T_A \, T_B$

Why?

$$T_{AB}(v) = \underline{ABv}$$

$$=$$

Properties of Matrix Multiplication

- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$
- $(B + C)A = BA + CA$
- $r(AB) = (rA)B = A(rB)$
- $I_m A = A = AI_n$, where I_m is the $m \times m$ identity matrix.

Multiplication is associative because function composition is.

Warning!

- AB is not always equal to
- $AB = AC$ does not mean that
- $AB = 0$ does not mean that