Section 2.3

Characterizations of Invertible Matrices

The Invertible Matrix Theorem

Suppose $A = n \times n$ matrix, and $T_A : \mathbb{R}^n \to \mathbb{R}^n$ is the associated linear transformation. The following are equivalent.

a) A is invertible

b) A is row equivalent to I_n

c) A has n pivots

d) $Ax = 0$ has only 0 solution

e) columns of A are linearly independent

f) T_A is one-to-one

g) $Ax = b$ is consistent for all b in $\mathbb{R}^n$

h) columns of A span $\mathbb{R}^n$

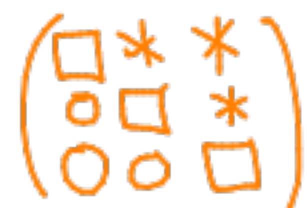
i) T_A is onto

j) A has a left inverse

k) A has a right inverse

l) A^T is invertible

$b \Longleftrightarrow c$



$BA = I_n$

$AB = I_n$

The Invertible Matrix Theorem

There are two kinds of square matrices, invertible (or non-singular), and non-invertible (or singular) matrices.

For invertible matrices, all of the conditions in the IMT hold. And for a non-invertible matrix, all of them fail to hold.

The Invertible Matrix Theorem

Poll

Which of the following are equivalent to the statement that A is invertible?

m) rows of A span $\mathbb{R}^n$

n) rows of A are linearly independent

o) $Ax = b$ has exactly one solution for all b in $\mathbb{R}^n$

p) $\det(A) \neq 0$, where $\det(A)$ is the volume of the parallelepiped formed by the columns of A

q) A^2 is invertible

Invertible Functions

A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is **invertible** if there is a function $g : \mathbb{R}^m \to \mathbb{R}^n$, so

$$f \circ g = g \circ f =$$

That is,

Fact. Suppose $A = n \times n$ matrix. Then T_A is invertible *as a function* if and only if . And in this case,

$$(T_A)^{-1} = T_{A^{-1}}$$

Why?