Announcements Feb 25

- Keeps tabs on your grades in TSquare
- WebWork 2.3 and 2.5 due Thursday
- Homework 5 due in class Friday (use a computer)
- Quiz 5 on 2.3 and 2.5 in class Friday
- Midterm 2 in class Friday Mar 11 on Chapters 2 & 3
- Office Hours Tuesday and Wednesday 2-3, after class, and by appt in Skiles 244 or 236
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 2.8

Subspaces of $\mathbb{R}^n$

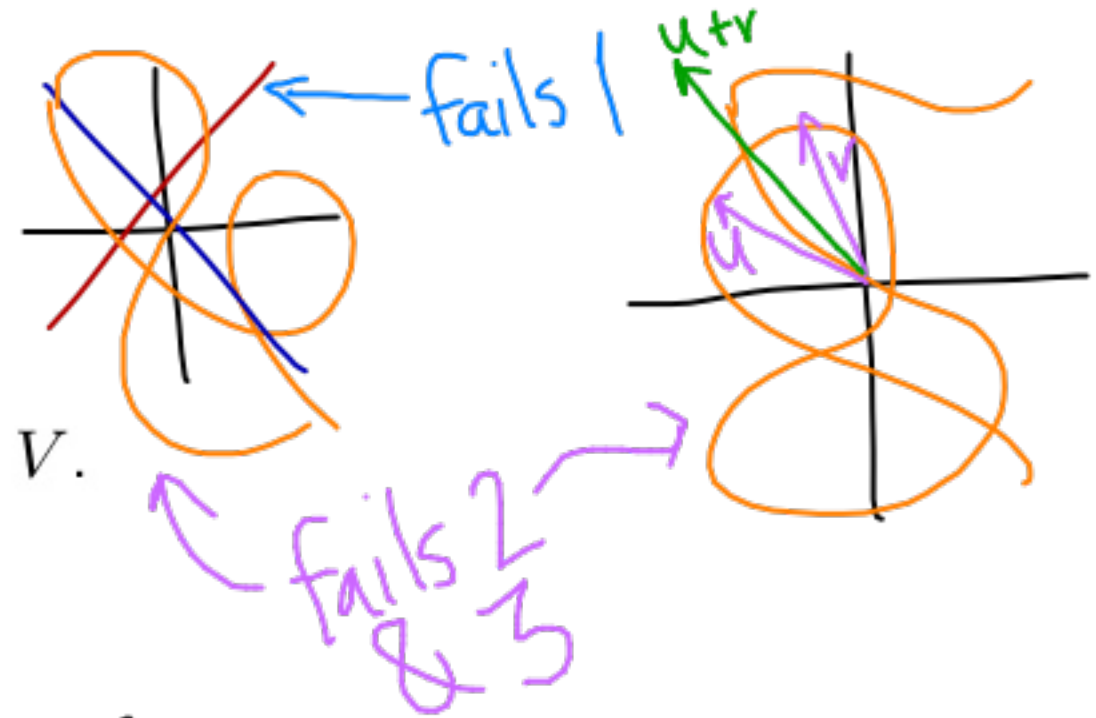
Subspaces
Outline

- Definition of subspace
- Examples and non-examples of subspaces
- Subspaces are the same as spans
- Bases for subspaces
- Two important subspaces for a matrix: $\mathrm{Col}(A)$ and $\mathrm{Nul}(A)$

Subspaces

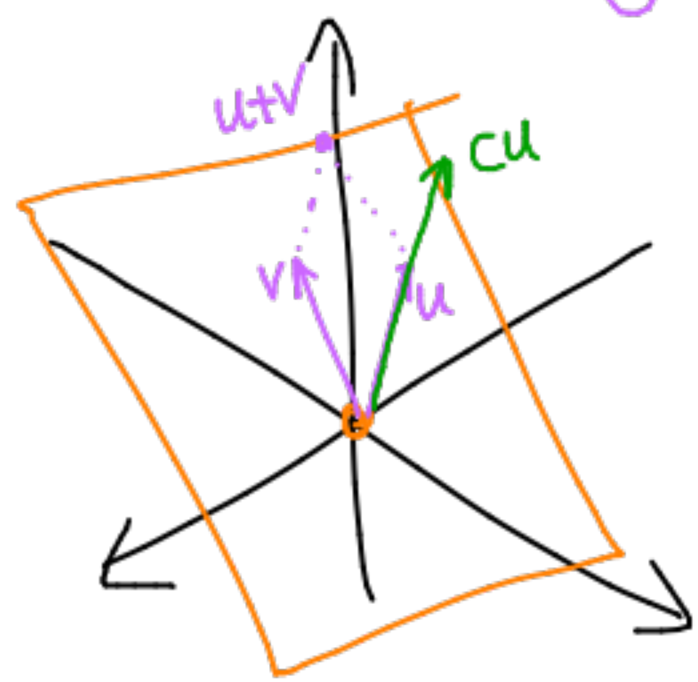
A subspace of $\mathbb{R}^n$ is a subset V of $\mathbb{R}^n$ with:

1. The zero vector is in V.
2. If u and v are in V, then $u + v$ is also in V.
3. If u is in V and c is in $\mathbb{R}$, then cu ~~is~~ in V.



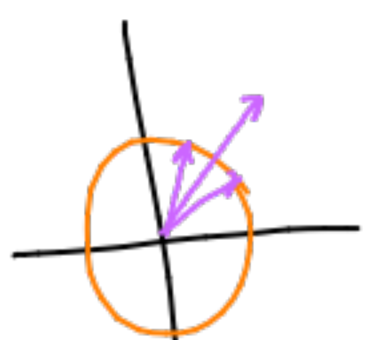
Examples

- line in $\mathbb{R}^n$ thru 0.
- plane in $\mathbb{R}^3$ thru 0. or $\mathbb{R}^n$
- $\mathbb{R}^n$



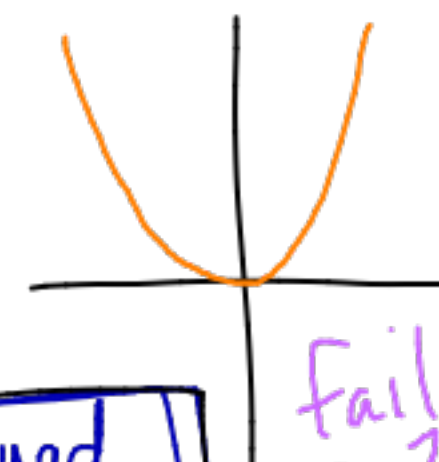
Non-examples

- circle:



fails 1-3

parabola



fails 2,3

-  1st & 3rd quad. Fails 2 only.

$\boxed{\text{1st quad. fails 3 only}}$

Spans are subspaces

Fact. Any $\mathrm{Span}\{v_1, \ldots, v_k\}$ is a subspace.

Why? Check 3 properties

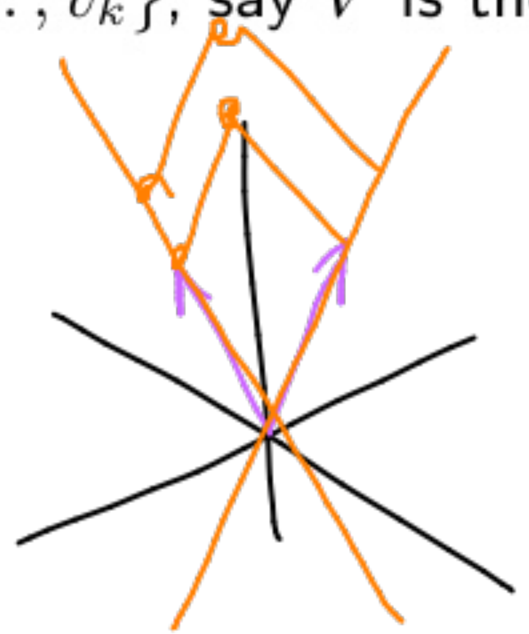
① take 0 lin. comb.

② $\underbrace{(3v_1+2v_2)}_{u} + \underbrace{(v_1+v_2)}_{v} = 4v_1+3v_2$ in $\mathrm{span}\{v_1, v_2\}$.

③ similar.

Note the following.

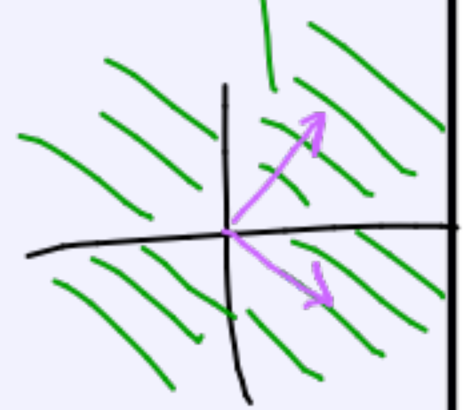
- If $V = \mathrm{span}\{v_1, v_2, \ldots, v_k\}$, say V is the subspace generated by $v_1, v_2, \ldots, v_k$.



Which are subspaces?

Poll

Which are subspaces? For those that are not subspaces, which part of the definition fails?

1. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a = 0 \right\}$ yes span $\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$
2. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a + b = 0 \right\}$ yes span $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$
3. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid ab = 0 \right\}$ no  fails 2
4. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid ab \neq 0 \right\}$ no  fail 1, 2, 3
5. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a, b \text{ rational} \right\}$ no fails 3

Subspaces are spans

Fact. Every subspace V is equal to some span.

Why?

$$V = \operatorname{span} V$$

We already said that all spans were subspaces, so now we know that three things are the same:

- spans
- subspaces
- planes thru 0


Vowel Sounds
Calendar & Weather
because
NASA

Column Space and Null Space

$A = m \times n$ matrix.

$\mathrm{Col}(A)$ = column space of A = span of the columns of A

$\mathrm{Nul}(A)$ = null space of A = (set of solutions to $Ax = 0$) ← a subspace! hence a span.

Check property 2: u, v solns to $Ax=0$

$A(u+v) = Au + Av$
$= 0 + 0 = 0$

Example. $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$

Then $\mathrm{Col}(A) = \mathrm{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$, line in $\mathbb{R}^3$

$\mathrm{Nul}(A) = \mathrm{span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$

$y = -x$

param form
$x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Column Space and Null Space

$A = m \times n$ matrix.

$\mathrm{Col}(A) =$ subspace of $\mathbb{R}^m$

$\mathrm{Nul}(A) =$ subspace of $\mathbb{R}^n$

Why? We checked on last slide.

Note that it is easier to check that $\mathrm{Nul}(A)$ is a subspace than it is to check that $\mathrm{Nul}(A)$ is a span.

Bases

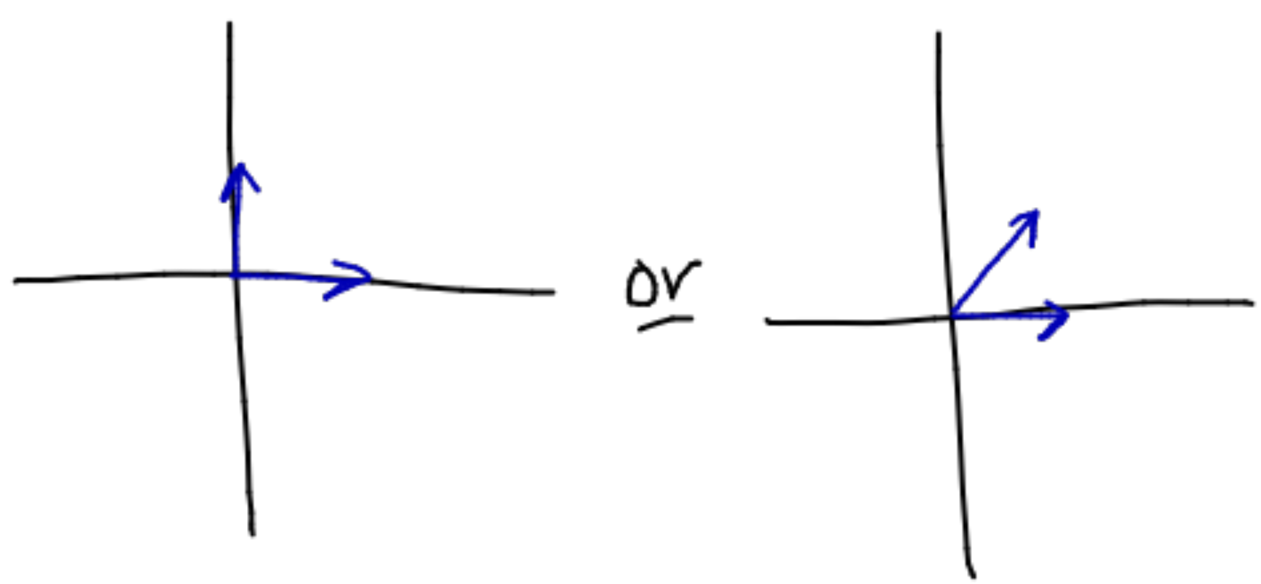
V = subspace of $\mathbb{R}^n$

A basis for V is a set of vectors $\{v_1, v_2, \ldots, v_k\}$ such that

1. $V = \text{span}\{v_1, \ldots, v_k\}$
2. the v_i are linearly independent

$\dim(V)$ = dimension of $V = k$ = # vectors in basis.

Q. What is one basis for $\mathbb{R}^2$? $\mathbb{R}^n$?



$\underline{\underline{\mathbb{R}^n}}$

$e_1, \ldots, e_n$

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

Q. Find bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

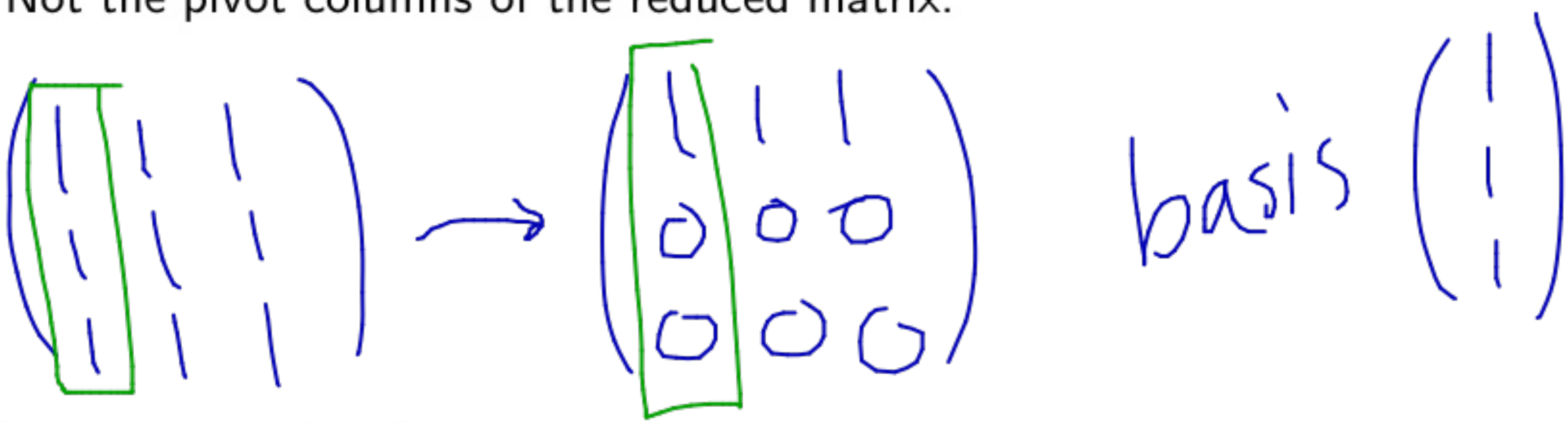
Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

In general:

- our usual parametric solution for $Ax = 0$ gives a basis for $\mathrm{Nul}(A)$
- the pivot columns of A form a basis for $\mathrm{Col}(A)$

$$x_2\begin{pmatrix}3\\1\\0\end{pmatrix} + x_3\begin{pmatrix}1\\6\\1\end{pmatrix}$$

Warning! Not the pivot columns of the reduced matrix.



Fact. If $A = n \times n$ matrix, then:

$$A \text{ is invertible} \Leftrightarrow \mathrm{Col}(A) = \mathbb{R}^n$$