

Announcements Feb 25

- Keeps tabs on your grades in TSquare
- WebWork 2.3 and 2.5 due Thursday
- Homework 5 due in class Friday (use a computer)
- Quiz 5 on 2.3 and 2.5 in class Friday
- Midterm 2 in class [Friday Mar 11 on Chapters 2 & 3](#)
- Office Hours Tuesday and [Wednesday](#) 2-3, after class, and by appt in Skiles 244 [or 236](#)
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
 - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
 - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
 - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 2.8

Subspaces of \mathbb{R}^n

Subspaces

Outline

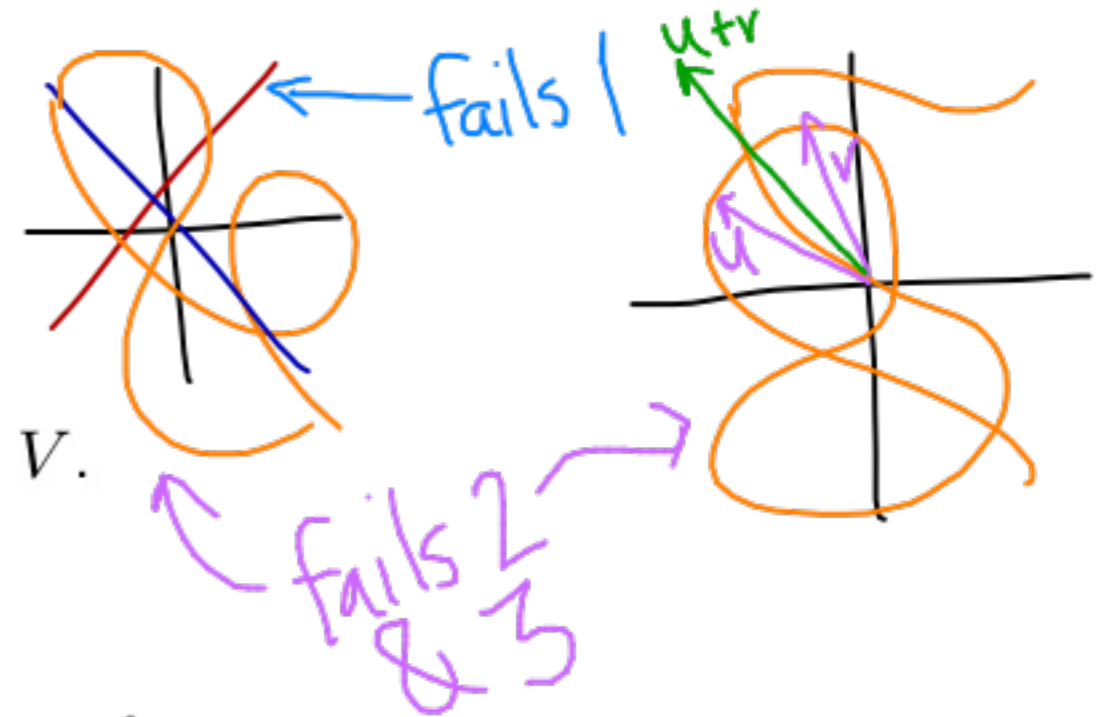
- Definition of subspace
- Examples and non-examples of subspaces
- Subspaces are the same as spans
- Bases for subspaces
- Two important subspaces for a matrix: $\text{Col}(A)$ and $\text{Nul}(A)$

Subspaces

A **subspace** of \mathbb{R}^n is a subset V with:

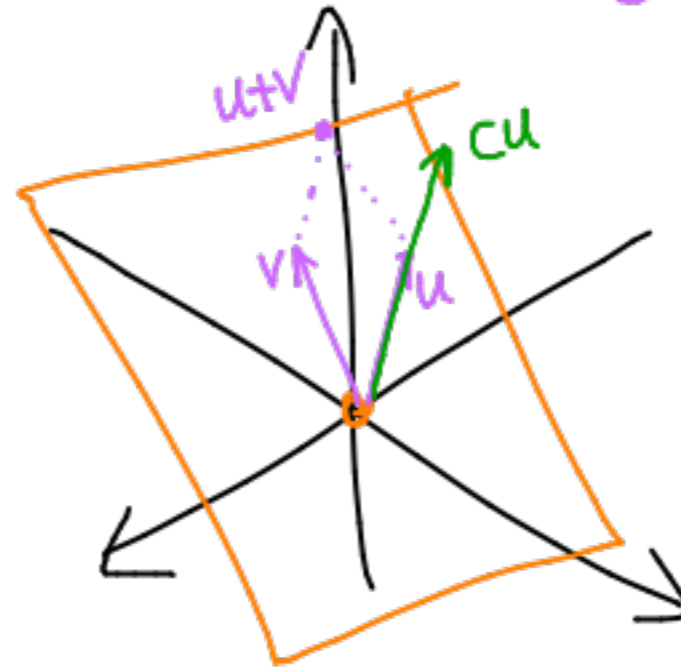
1. The zero vector is in V .
2. If u and v are in V , then $u + v$ is also in V .
3. If u is in V and c is in \mathbb{R} , then cu is in V .

of \mathbb{R}^n



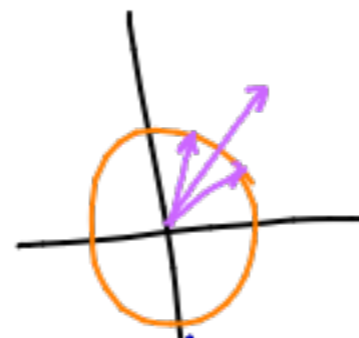
Examples

- line in \mathbb{R}^n thru 0.
- plane in \mathbb{R}^3 thru 0.
- \mathbb{R}^n or \mathbb{R}^n



Non-examples

- circle
- 1st & 3rd quad.



fails 1-3

fails 2 only.

parabola

1st quad.
fails 3 only



fails 2, 3

Spans are subspaces

Fact. Any $\text{Span}\{v_1, \dots, v_k\}$ is a subspace.

Why? Check 3 properties

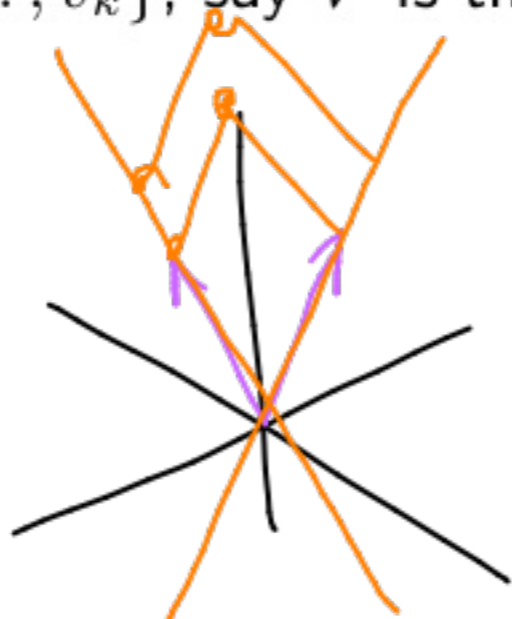
① take 0 lin. comb.

② $(3v_1 + 2v_2) + (v_1 + v_2) = 4v_1 + 3v_2$
in $\text{span}\{v_1, v_2\}$.

③ Similar.

Note the following.

- If $V = \text{span}\{v_1, v_2, \dots, v_k\}$, say V is the subspace **generated by** v_1, v_2, \dots, v_k .



Which are subspaces?

Poll

Which are subspaces? For those that are not subspaces, which part of the definition fails?

1. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a = 0 \right\}$ yes

span $\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

2. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a + b = 0 \right\}$ yes

span $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$

3. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid ab = 0 \right\}$ no

fails 2

4. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid ab \neq 0 \right\}$ no

fail 1, 2, 3

5. $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a, b \text{ rational} \right\}$ no

fails 3

Subspaces are spans

Fact. Every subspace V is equal to some span.

Why?

$$V = \text{span } V$$

We already said that all spans were subspaces, so now we know that three things are the same:

- spans
- subspaces
- planes thru 0



Vowel Sounds

a	apple
e	egg
i	ice cream
o	orange
u	umbrella

Mixed Sounds

ch	chicken
sh	ship
th	thumb
ph	phone
ck	clock
ng	king
gh	ghost
qu	queen
wh	wheel
ll	lily
ss	sun
tt	top
pp	pop
ff	fish
mm	moon
nn	nose
zz	zoo
xx	xylophone
yy	yo-yo
vv	van
ww	wheel
bb	ball
dd	door
gg	goat
kk	kite
ll	lily
mm	moon
nn	nose
oo	orange
pp	pop
qq	queen
rr	rain
ss	sun
tt	top
uu	umbrella
vv	van
ww	wheel
xx	xylophone
yy	yo-yo
zz	zoo

Work My Words

Write the words in the boxes.

1. cat
2. dog
3. pig
4. cow
5. sheep
6. horse
7. chicken
8. turkey
9. duck
10. frog
11. bear
12. rabbit
13. mouse
14. snake
15. spider
16. butterfly
17. bee
18. ant
19. worm
20. ladybug

because

1. One vowel
2. closed in by
a consonant

Short vowel
Squash

ar
or
er
ir

1. Be Alert
2. LISTEN to the person talking.



Calendar Weather

Month: _____

Day: _____

Weather: _____

Vowel Traps

ea
ai
oi
ou

Crain do you



Column Space and Null Space

$A = m \times n$ matrix.

$\text{Col}(A) =$ **column space** of $A =$ span of the columns of A

$\text{Nul}(A) =$ **null space** of $A =$ (set of solutions to $Ax = 0$) \leftarrow a subspace!
hence a span.

Example. $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$

Then $\text{Col}(A) = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$, line in \mathbb{R}^3

$\text{Nul}(A) = \text{span} \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$

$y = -x$

Check property 2:
 u, v solns to
 $Ax = 0$

$A(u+v) = Au + Av$
 $= 0 + 0 = 0$

param form
 $x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Column Space and Null Space

$A = m \times n$ matrix.

$\text{Col}(A) =$ subspace of \mathbb{R}^m

$\text{Nul}(A) =$ subspace of \mathbb{R}^n

Why? We checked
on last slide.

Note that it is easier to check that $\text{Nul}(A)$ is a subspace than it is to check that $\text{Nul}(A)$ is a span.

Bases

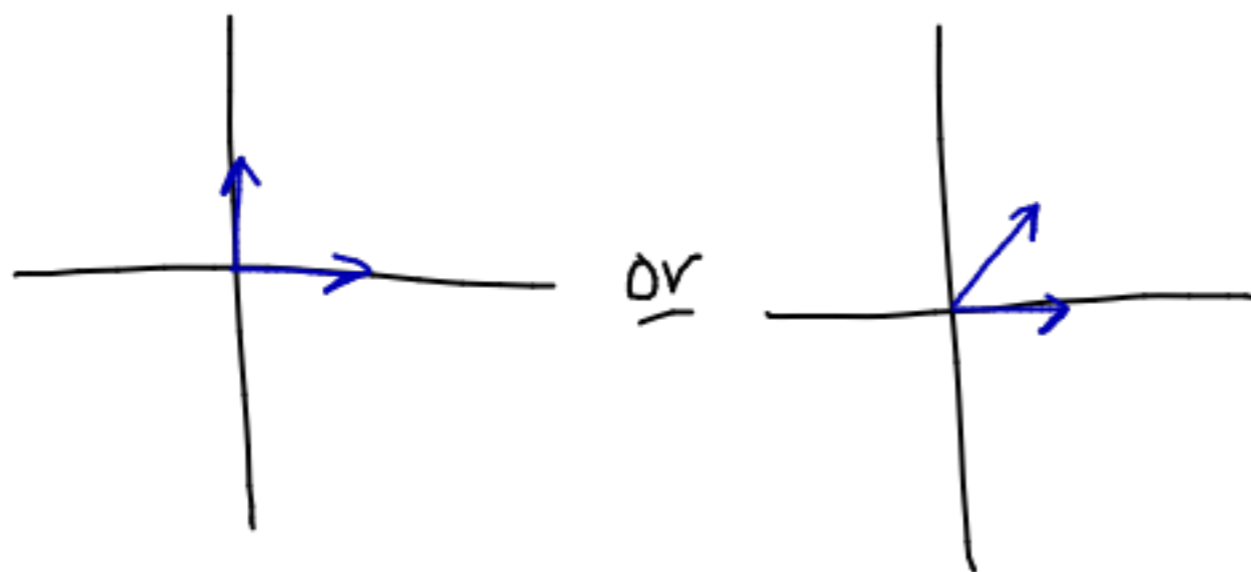
$V =$ subspace of \mathbb{R}^n

A **basis** for V is a set of vectors $\{v_1, v_2, \dots, v_k\}$ such that

1. $V = \text{span}\{v_1, \dots, v_k\}$
2. the v_i are linearly independent

$\dim(V) =$ **dimension** of $V = k = \#$ vectors in basis.

Q. What is one basis for \mathbb{R}^2 ? \mathbb{R}^n ?



\mathbb{R}^n
 e_1, \dots, e_n

Bases for $\text{Nul}(A)$ and $\text{Col}(A)$

Q. Find bases for $\text{Nul}(A)$ and $\text{Col}(A)$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Bases for $\text{Nul}(A)$ and $\text{Col}(A)$

In general:

- our usual parametric solution for $Ax = 0$ gives a basis for $\text{Nul}(A)$
- the pivot columns of A form a basis for $\text{Col}(A)$

$$x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix}$$

Warning! Not the pivot columns of the reduced matrix.

$$\begin{pmatrix} | & | & | \\ | & | & | \\ | & | & | \\ | & | & | \end{pmatrix} \rightarrow \begin{pmatrix} | & | & | \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

basis $\begin{pmatrix} | \\ | \\ | \end{pmatrix}$

Fact. If $A = n \times n$ matrix, then:

$$A \text{ is invertible} \Leftrightarrow \text{Col}(A) = \mathbb{R}^n$$