Announcements Feb 5

- Lecture Notes are online!
- Homework 3 due now; circle your info at top right, pass to aisle and forward
- Quiz 3 today on Section 1.5
- Midterm 1 in class next week Friday Feb 12 on Chapter 1
- My Office Hours Tuesday and Wednesday 2-3, after class, and by appointment
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12
    - LA hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Skiles 244

Section 1.7: Linear Independence

Quick summary

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0 \ldots$$ only 0 soln.

$$\begin{pmatrix} v_1 & \cdots & v_k \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} = 0$$

The columns of an $m \times n$ matrix A are linearly independent $\Leftrightarrow$ A has...

pivot in each col.
$\Rightarrow Ax = 0$ has only 0 soln

To check if $\{v_1, \ldots, v_k\}$ is linearly independent check if each v_i is...

not in span$\{v_1, \ldots, v_{i-1}\}$

Fact. Say $v_1, \ldots, v_k$ are in $\mathbb{R}^n$. If $k > n$, then $\{v_1, \ldots, v_k\}$ is

dependent

Q1. *True/False.* If three vectors span $\mathbb{R}^3$ then those three vectors must be linearly independent.

Q2. Which of the following true statements can be checked without row reduction?

1. $\{(3,3,4),(0,0,\pi),(0,\sqrt{2},0)\}$ is linearly independent
2. $\{(3,3,4),(0,10,20),(0,5,7)\}$ is linearly independent
3. $\{(3,3,4),(0,10,20),(0,5,7),(0,0,1)\}$ is linearly dependent
4. $\{(3,3,4),(0,10,20),(0,0,0)\}$ is linearly dependent

More than one!

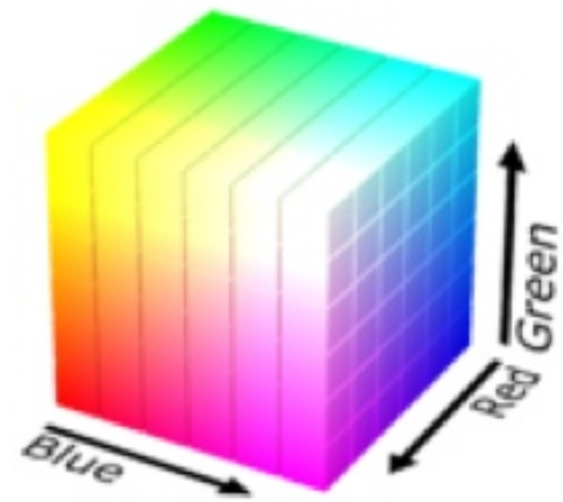
$$0\begin{pmatrix}3\\3\\4\end{pmatrix}+0\begin{pmatrix}0\\10\\20\end{pmatrix}+25\begin{pmatrix}0\\0\\0\end{pmatrix}=0 \qquad \begin{pmatrix}3&0&0\\3&0&\sqrt{2}\\4&\pi&0\end{pmatrix}$$

Q3. How many solutions can there be to $Ax=b$ if the columns of A are linearly independent?

1. 0
2. 1
3. ∞

$$\begin{pmatrix}1&1\\1&0\\1&1\end{pmatrix}x=\begin{pmatrix}1\\2\\3\end{pmatrix} \qquad a\begin{pmatrix}1\\1\\1\end{pmatrix}+b\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}a+b\\a\\a+b\end{pmatrix}$$

Every color is a vector in $\mathbb{R}^3$ with coordinates between 0 and 256. The three coordinates correspond to red, green, and blue.



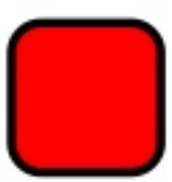
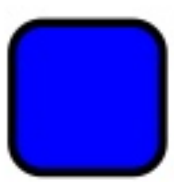
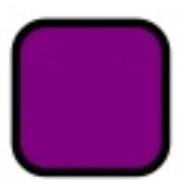
Given colors $v_1, \ldots, v_k$, we can form a new color by making a linear combination

$$c_1 v_1 + \cdots + c_k v_k$$

where $c_1 + \cdots + c_k = 1$

$$\frac{1}{2}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1/2\\0\\1/2\end{pmatrix}$$

Example:

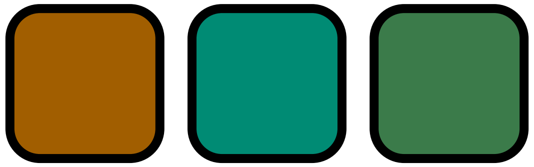
$\frac{1}{2}$  $+$ $\frac{1}{2}$  $=$ 

Section 1.7: Linear Independence

Application: Additive Color Theory

Consider now the three colors

$$\begin{pmatrix} 240 \\ 140 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 120 \\ 100 \end{pmatrix}, \begin{pmatrix} 60 \\ 125 \\ 75 \end{pmatrix}$$



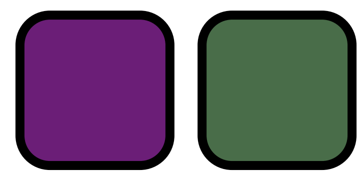
Are these colors linearly independent? What does your answer tell you about the colors?

Section 1.7: Linear Independence

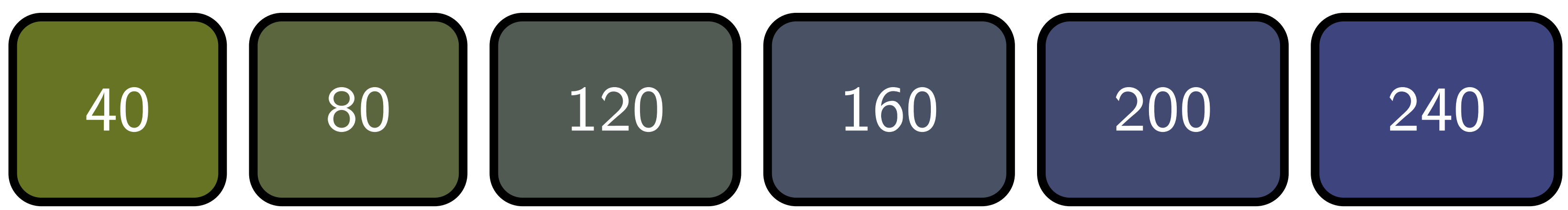
Application: Additive Color Theory

Consider now the two colors

$$\begin{pmatrix} 180 \\ 50 \\ 200 \end{pmatrix}, \begin{pmatrix} 100 \\ 150 \\ 100 \end{pmatrix}$$



For which h is $(116, 130, h)$ in the span of those two colors?



Section 1.8: Linear Transformations

Quick summary

Given an $m \times n$ matrix A we define a function

$$T_A : \mathbb{R}^n \to \mathbb{R}^m$$
$$T_A(v) =$$

The domain of T_A is
The co-domain/target of T_A is
The range/image of T_A is

When $m = n$ we can think of T_A as doing something to $\mathbb{R}^n$.

A function $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear if

- $T(u + v) =$
- $T(cv) =$

Fact. Every matrix transformation T_A is linear.

Q1. Say A is a $1 \times n$ matrix. If $T_A(v) = 3$ and $T_A(w) = -1$, then what is

$$T_A(7v - 5w)?$$

Q2. Find a 3×3 matrix A so that $T_A(v) = v$ for all v in $\mathbb{R}^3$.

Q3. Say A is an $m \times 2$ matrix. If the columns of A are linearly independent, what does the image of T_A look like geometrically?

For each matrix A, describe what T_A does to $\mathbb{R}^3$.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$