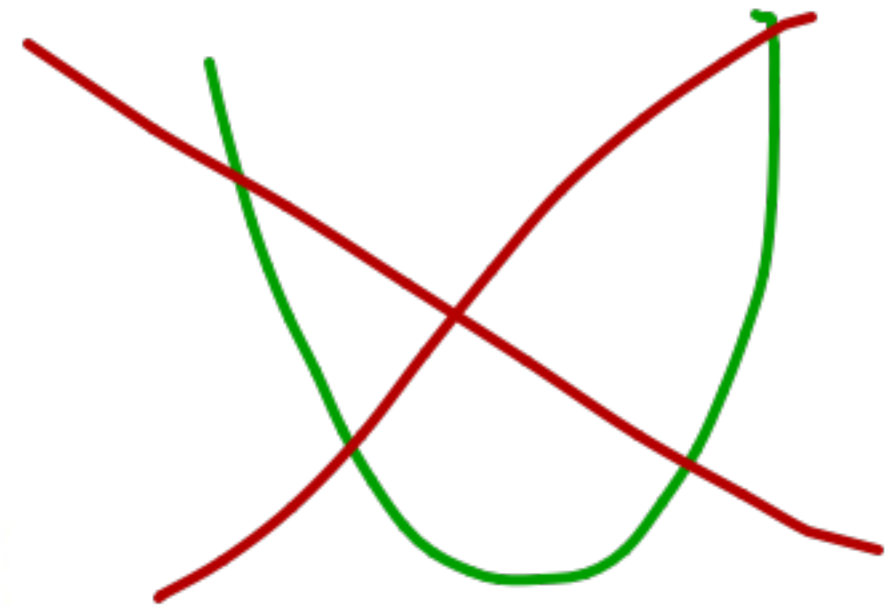
$5x-3y$

Chapter 1



Linear Equations

Section 1.1

Systems of Linear Equations

$3x - y = 5$

Systems of Linear Equations

The solution to a single linear equation can be...

$\emptyset$ line pt. plane. higher-dim. plane.

$x+y+z=1$

The solution to **system** of linear equations is...

intersection of these.

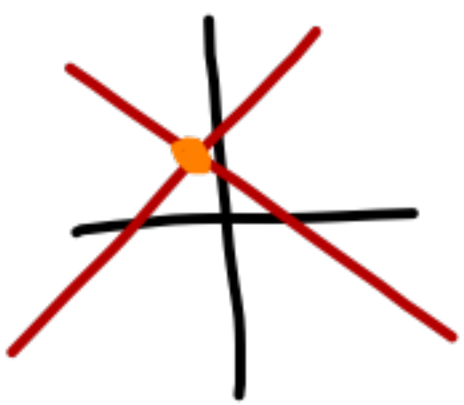
For example, consider this system.

$$x - 3y = -3$$
$$2x + y = 8$$

$\rightarrow$

$$-2x+6y=6$$
$$\underline{2x+y=8}$$
$$7y=14$$
$$y=2 \rightarrow x=3$$

elim.



other method: subst.

Example

Solve:

$$\begin{aligned} -3(x + 2y + 3z &= 6) \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

How many ways can you do it?

elimin.

subst.

Example

Solve:

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

In what ways can you manipulate the equations?

scale an eqn

add two eqns (*)

Example

Solve:

$$
\begin{aligned}
x + 2y + 3z &= 6 \\
2x - 3y + 2z &= 14 \\
3x + y - z &= -2
\end{aligned}
$$

It is redundant to write x, y, z again and again, so we rewrite using *matrices*:

$$
\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -3 & 2 & 14 \\ 3 & 1 & -1 & -2 \end{array}\right) \longrightarrow
\left(\begin{array}{cccc} -3 & -6 & -9 & -18 \\ 2 & -3 & 2 & 14 \\ 3 & 1 & -1 & -2 \end{array}\right) \rightsquigarrow
\left(\begin{array}{ccc|c} -3 & -6 & -9 & -18 \\ 2 & -3 & 2 & 14 \\ 0 & -5 & -10 & -20 \end{array}\right)
$$

$$
\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 2 & -3 & 2 & 14 \end{array}\right) \rightsquigarrow
\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & -7 & -4 & 2 \end{array}\right) \rightsquigarrow
\left(\begin{array}{cccc} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 10 & 30 \end{array}\right)
$$

$10z = 30$
$z = 3$

$y + 2z = 4$
$y + 6 = 4$
$y = -2$

$x + 2y + 3z = 6$
$x - 4 + 9 = 6$
$x = 1$

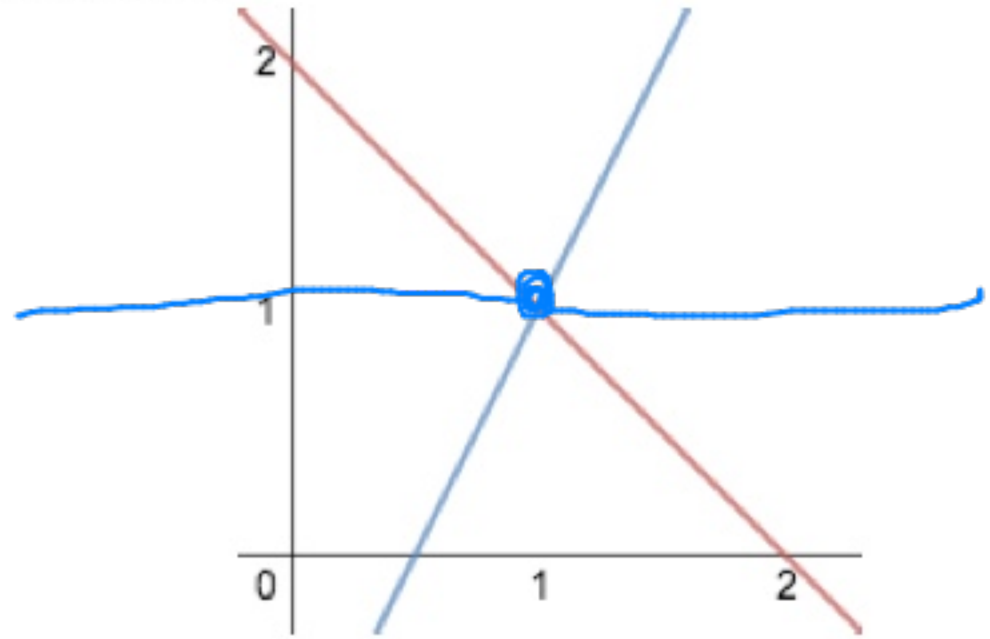
$(1, -2, 3)$

Row operations

Our manipulations of matrices are called row operations.
Why do row operations not change the solution?
Solve:

$$x + y = 2$$
$$-2x + y = -1$$

System has one solution, $x = 1, y = 1$.



What happens to the two lines as you do row operations?

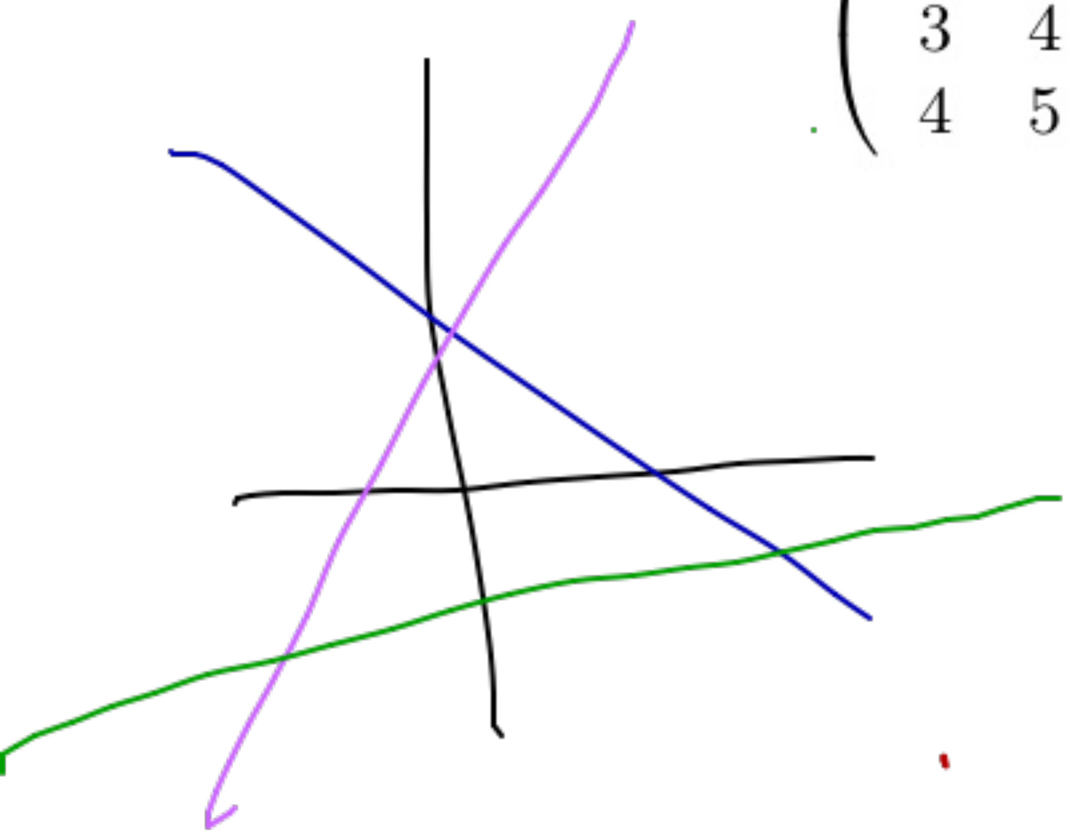
$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ -2 & 1 & -1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 3 & 3 \end{array}\right)$$

A Bad Example

Solve:

$$\begin{aligned} x + y &= 2 \\ 3x + 4y &= 5 \\ 4x + 5y &= \cancel{7}\,9 \end{aligned}$$

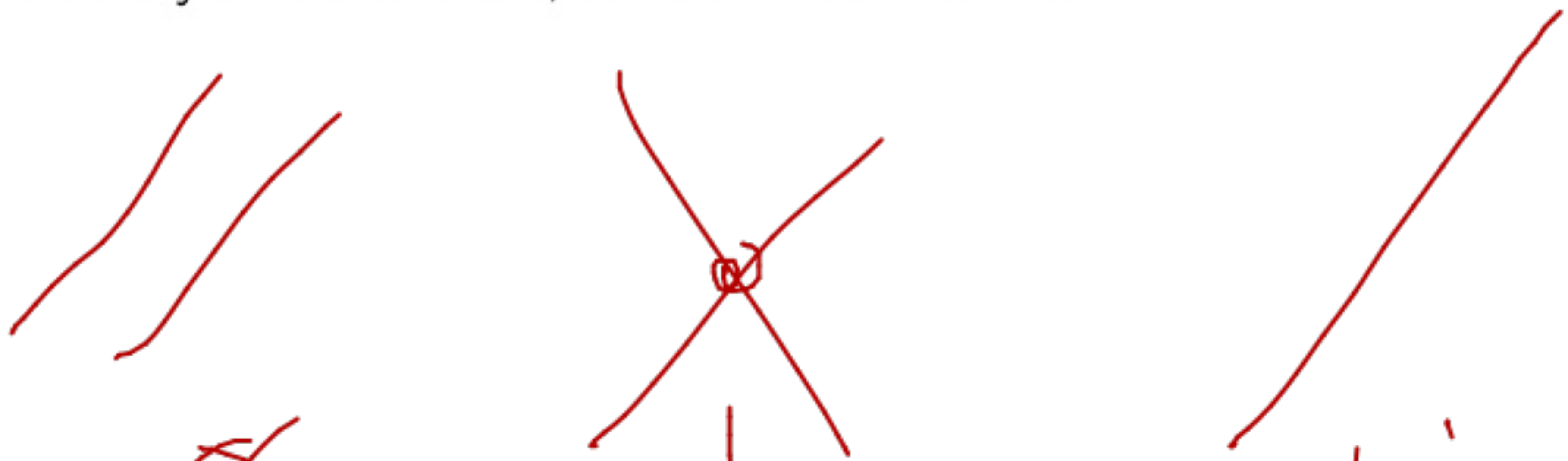
$$\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 4 & 5 \\ 4 & 5 & 9 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{array}\right)$$



We say the system is... inconsistent

What can the solution look like?

With only two variables, the solution can be...



$\emptyset$ pt line

What happens when we have three variables?

Poll

In how many ways can 3 planes intersect in $\mathbb{R}^3$?