Announcements Jan 20

- WebWork 1.1 and 1.2 due Thursday at 11:59 pm
- Homework 1 due Friday at start of class
- Quiz 1 in class Friday on Sections 1.1-1.4
- Midterm 1 in class Friday Feb 12
- Office Hours Tuesday and Wednesday 2-3
- LA Office Hours: Scott Mon 12-1, Yashvi Mon 2-3, Baishen Wed 4-5, Matt Thu 3-4, Shivang Fri 10:30-11 + 12:30-1
- Math Lab, Clough 280
    - Regular hours: Mon/Wed 11-5 and Tue/Thu 11-5
    - Math 1553 hours: Mon-Thu 5-6 and Tue/Thu 11-12.
    - LA Math Lab hours: Matt Tue 11-12, Scott Tue 5-6, Baishen Thu 11-12, Yashvi/Shivang Thu 5-6

Section 1.3

Vector Equations

Vectors

A *vector* is a matrix with one row or one column.

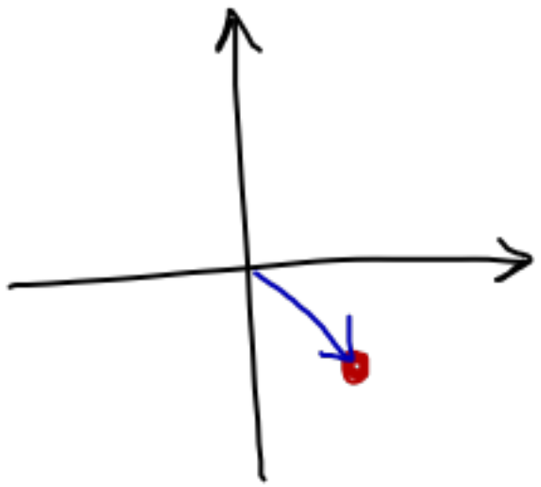
$$\begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 4 & 1 & -3 \end{pmatrix}$$

Adding vectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ "add component-wise"

Scaling vectors $7 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \end{pmatrix}$ "scale component-wise"

Geometry

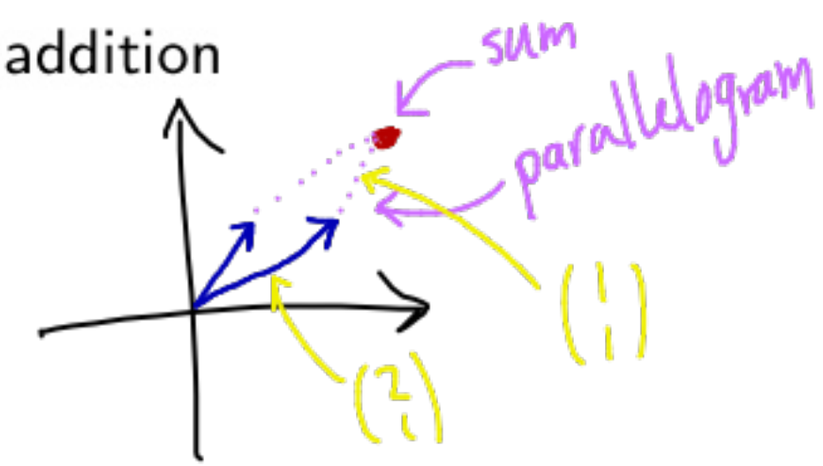
A length n vector can be drawn as a point or arrow in $\mathbb{R}^n$.



$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

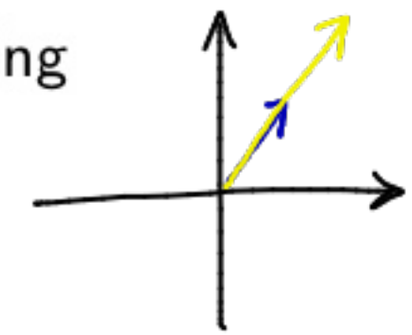
Parallelogram rule for addition

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



Scaling

$2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



Linear Combinations

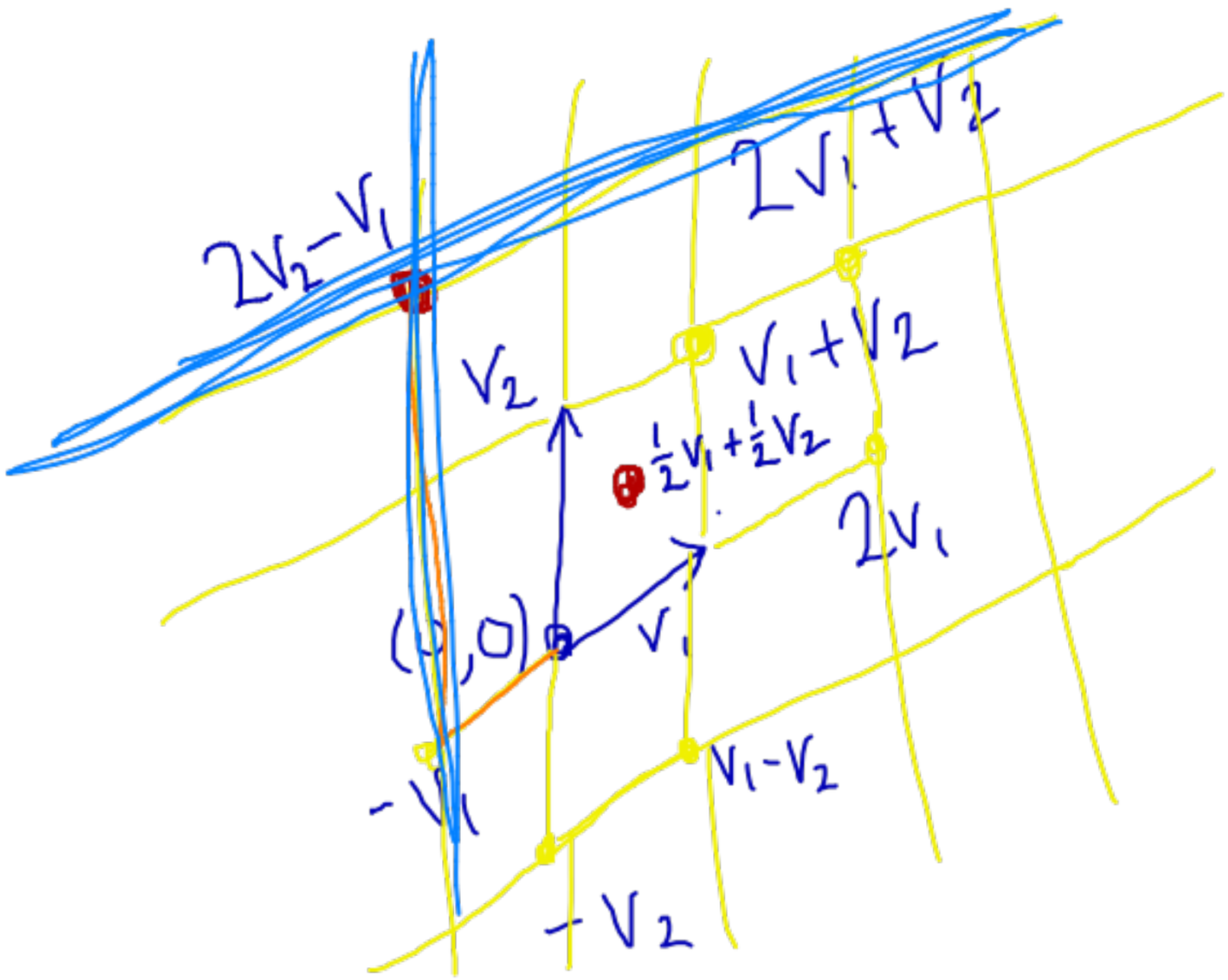
A *linear combination* of the vectors $v_1, v_2, \cdots, v_k$ is

$c_1v_1 + c_2v_2 + \cdots + c_kv_k$

$c_1, \ldots, c_k$ real numbers

in $\mathbb{R}^n$

a new vector



Linear Combinations

Q. Is $\begin{pmatrix} 8 \\ 16 \\ 3 \end{pmatrix}$ a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$?

$\rightsquigarrow$ solve: $a\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} + b\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \\ 3 \end{pmatrix}$

$\rightsquigarrow$ solve:

$$a - b = 8$$
$$2a - 2b = 16$$
$$6a + b = 3$$

$\rightsquigarrow$ row reduce:

$$\left(\begin{array}{cc|c} 1 & -1 & 8 \\ 2 & -2 & 16 \\ 6 & 1 & 3 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -1 & 8 \\ 0 & 0 & 0 \\ 0 & 7 & -45 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & 8 - 45/7 \\ 0 & 1 & -45/7 \\ 0 & 0 & 0 \end{array}\right)$$

yes

$\rightsquigarrow$ $a = 11/7$, $b = -45/7$

Span

$$\begin{aligned}\text{Span}\{v_1, v_2, \ldots, v_k\} &= \{c_1v_1 + c_2v_2 + \cdots + c_kv_k \mid c_i \text{ in } \mathbb{R}\} \\ &= \text{~~the set of~~ all linear combinations of vectors } v_1, v_2, \ldots, v_k \\ &= \text{plane through the origin and } v_1, v_2, \ldots, v_k.\end{aligned}$$

the set of

so that

take all $c_i = 0$.

We just saw: the question of whether u is in Span$\{v_1, v_2, \ldots, v_k\}$, is equivalent to solving a linear system by row reducing the matrix

Application

Consider the production costs:

	Materials	Labor	Overhead
Widget	\$1	\$2	\$3
Gadget	\$4	\$5	\$6

Q. What are possible expenditures on materials, labor, and overhead?

$$\text{span}\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}\right\}$$

Poll

If u is a linear combination of $v_1, v_2, \cdots v_K$, then there is only one way to write u as (that is, there is only one choice for $c_1, c_2, \ldots, c_k$).

- true
- false

$$1\cdot\begin{pmatrix}1\\1\end{pmatrix}+2\cdot\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}5\\5\end{pmatrix}$$

$$3\cdot\begin{pmatrix}1\\1\end{pmatrix}+1\cdot\begin{pmatrix}2\\2\end{pmatrix}=$$

$$3\cdot\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$1\cdot\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$