Chapter 2

Review

Products

$AB \neq BA$

$T_{AB} = T_A \circ T_B$

Inverses

- if $AB = BA = I$, then $B = A^{-1}$
- $T_A^{-1} = T_{A^{-1}}$
- $Ax = b \rightsquigarrow x = A^{-1}b$
- Find A^{-1} by row reducing $(A|I)$ to get $(I|A^{-1})$
- Another perspective: $E_k E_{k-1} \cdots E_2 E_1 A = I$, where each E_i is an elementary matrix. This implies that $E_k E_{k-1} \cdots E_2 E_1 = A^{-1}$.

Chapter 2 Review

Invertible Matrix Theorem

Let A be an $n \times n$ matrix. The following are equivalent:

- A is invertible
- A can be reduced to I
- A has n pivots
- $\mathrm{Nul}(A) = \{0\}$
- T_A is one-to-one
- T_A is onto
- $Ax = b$ is consistent for all b
- $\dim \mathrm{Col}(A) = n$

etc ...

Chapter 2 Review

LU Decompositions

Let $A = LU$, where L is lower triangular, U is in echelon form.

- Easy to solve for many b.
- Algorithm
  - Step 1. Solve $Ly = b$
  - Step 2. Solve $Ux = y$
- To find L and U, row reduce A to echelon form to obtain U, then use negative of row operations to obtain L. Be careful:
  - go column by column
  - only use lower row replacement

Chapter 2 Review

Subspaces

- A subspace is a non-empty subset of $\mathbb{R}^n$ closed under linear combinations.
- Two important subspaces are
  - $\mathrm{Col}(A) =$ span of columns of A.
  - $\mathrm{Nul}(A) =$ (solutions to $Ax = 0$).
- A basis for a subspace W is a set of lin. ind. vectors that spans W.
  - To find the B–coords of u, solve $Bx = u$
- The dimension of a subspace is the number of elements in the basis.
- Use row reduction to find a basis for Col(A) or Nul(A).
  - Pivot columns are used to identify the basis for Col(A).
  - Parametric form is used to identify the basis for Nul(A).

Note. $\mathrm{Col}(A) =$ range of T_A

Chapter 2 Review

Rank Theorem Let A be an $m \times n$ matrix, then

$$\dim \mathrm{Col}(A) + \dim \mathrm{Nul}(A) = n.$$

Basis Theorem

Suppose V is a k-dimensional subspace of $\mathbb{R}^n$. Then

- Any k linearly independent vectors in V form a basis for V.
- Any k vectors in V that span V form a basis.

Midterm 2

Piazza Questions

Matrix Algebra

Solve for X:

$$(A - AX)^{-1} = X^{-1}B$$

Matrix Algebra

True/False.

$(A + B)^2 = A^2 + 2AB = B^2$

If A and B are invertible then $A + B$ is invertible.

If $Ax = 0$ has the trivial solution then A is invertible.

Elementary Matrices

Find a product of elementary matrices that equals

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

Areas and Determinants

Find the area of the triangle in $\mathbb{R}^2$ with vertices $(-5,3)$, $(0,5)$, and $(47,24)$.

Let B be a ball of radius 1 in $\mathbb{R}^3$. The volume of B is $4\pi/3$. Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 9 \\ 0 & 0 & 2 \end{pmatrix}$$

What is the volume of $T_A(B)$?

Inverses and Determinants

Find the matrix of cofactors for

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

What is A^{-1}?

Properties of Determinants

Suppose that A is a 5×5 matrix with determinant 3.

Is A invertible?

What is $\det(-A)$?

What is $\det(A^{-1})$?

What is $\det A^T$?

What is the determinant of the matrix obtained from A by replacing the first row with twice the first row plus the second row?

Bases and Coordinates

Let

$$b_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } b_2 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

and $B = \{b_1, b_2\}$. Explain why B is a basis for $\mathbb{R}^2$.

What is $[e_1]_B$? In other words, what are the B–coordinates of $e_1 = (1, 0)$?

Computing Determinants

Compute:

$$\det \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$