Scores: 1 2 3 4 5

Name _____

Section ____

Mathematics 1553

Practice Midterm 1

Prof. Margalit

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and let T_A be the associated linear transformation.

What is the domain of T_A?

Is T_A one-to-one?

Is T_A onto?

What is the dimension of the set of solutions to $Ax = 0$ (that is, how many free variables)?

Does $Ax = b$ have a solution for every b in $\mathbb{R}^3$?

What is the span of the columns of A?

(a) the xy–plane

(b) the yz–plane

(c) the xz–plane

(d) none of the above

2. *(a)* Find a matrix so that the associated linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ is reflection about the line $y = -x$.

Find three vectors u, v, and w in $\mathbb{R}^3$ so that u and v are linearly independent but u, v, and w are linearly dependent.

If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto, what can you say about m and n? Choose one answer.

(a) $m < n$

(b) $m \leq n$

(c) $m = n$

(d) $m \geq n$

(e) $m > n$

(f) I cannot say any of these definitively

(b) Consider the linear system

$$\begin{aligned} x + y &= 1 \\ x + ky &= b \end{aligned}$$

Find a k and a b so that the system is inconsistent.

Find a k and a b so that the system has infinitely many solutions.

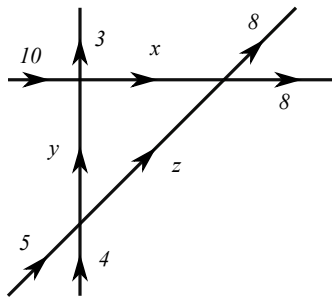
Find a k and a b so that the system has exactly one solution.

3. Complete ONE of the following two problems. Circle the letter of the one you chose.

(a) Determine all values of x so that the vectors $(1, 1, x)$, $(1, x, 1)$, and $(x, 1, 1)$ are linearly dependent. Explain your answer.

(b) For which values of h is $(1, 2, h)$ in the span of $(1, 1, 2)$ and $(1, 1, 3)$? Explain your answer.

4. The following diagram indicates traffic flow in one part of town:



Write a system of linear equations in x, y, and z describing the traffic flow around the triangle.

Turn the system of linear equations from the first part into an augmented matrix A.

Copy your matrix A from the previous page. Find the reduced row echelon form of A.

Write down the set of solutions of the original system of linear equations.

Write the set of solutions to the original system of linear equations in parametric form.

What is the maximum amount of traffic on the road labeled z?

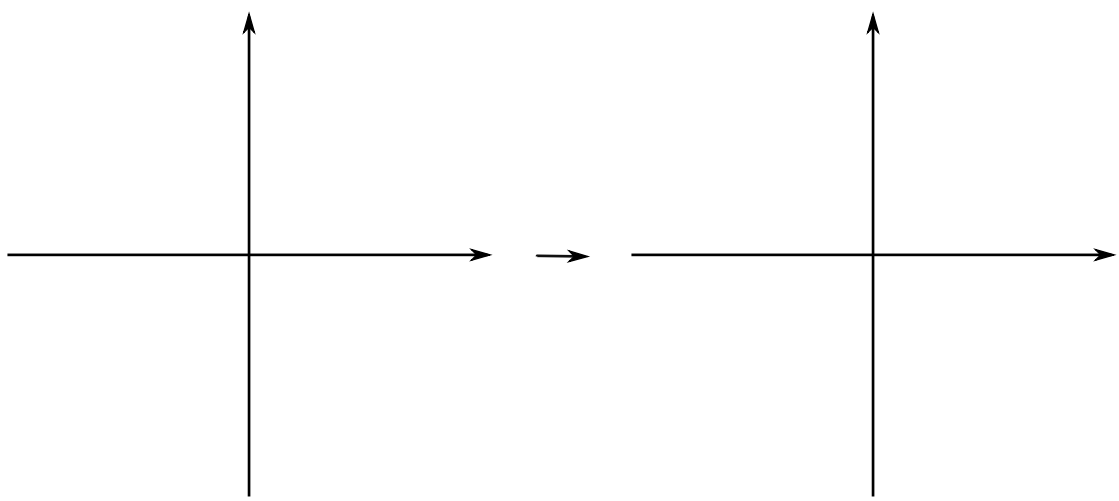
5. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Let T_A be the associated linear transformation.

What is $T_A(100, 1)$?

On the right-hand side draw the range of T_A. On the left-hand side draw the set of points v in the domain with $T_A(v) = 0$.



The picture on the left shows:

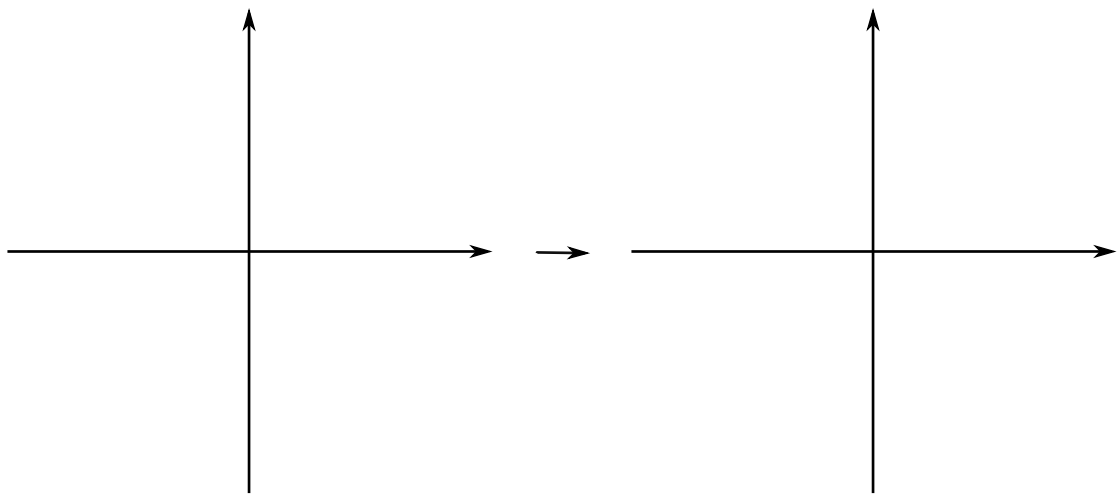
(a) The solutions to $Ax = 0$

(b) The product of A with the 0 vector

(c) The solutions to $Ax = b$ for all b

(d) None of the above

On the right-hand side draw the range of T_A again. Choose any nonzero point a in the image of T_A and label it with its (x, y)–coordinates. On the left-hand side draw the set of points v in the domain with $T_A(v) = a$.



Is there a vector b in $\mathbb{R}^2$ so that the solutions to $Ax = b$ is the line $x + y = 6$? If so, find such a b; if not, explain why not.

Is there a vector b in $\mathbb{R}^2$ so that the solutions to $Ax = b$ is the line $x - y = 6$? If so, find such a b; if not, explain why not.