1. Consider the matrix

\[ A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \]

Find a basis for the column space of \( A \).

What is the dimension of the column space of \( A \)?
2. Consider again the matrix

\[ A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \]

Find a basis for the null space of \( A \).

What is the dimension of the null space of \( A \)?
3. Let 
\[ b_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } b_2 = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \]
and \( B = \{b_1, b_2\} \). Explain why \( B \) is a basis for \( \mathbb{R}^2 \).

What is \([e_1]_B\)? In other words, what are the \( B \)-coordinates of \( e_1 = (1, 0) \)?
4. Consider the set of vectors

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

with \(xyz = 0\). Do these vectors form a subspace of \(\mathbb{R}^3\)? Explain your answer.

Suppose that \(A\) is a \(5 \times 2\) matrix and that the image of the associated linear transformation is a 2–dimensional plane. Describe the set of solutions to \(Ax = 0\). Explain your answer.
5. Find an LU factorization of the following matrix:

\[ A = \begin{pmatrix} 5 & -1 & 2 & 3 \\ 10 & 3 & 7 & 6 \\ 0 & 20 & 13 & 5 \end{pmatrix} \]
6. The matrix

\[ A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \]

has the LU factorization \( A = LU \) where

\[ L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}. \]

Use this factorization to solve

\[ Ax = \begin{pmatrix} 1 \\ -4 \end{pmatrix}. \]

Show clearly the two steps.
7. Let $A$ be an $n \times n$ matrix. Which of the following statements are equivalent to the statement “$A$ is invertible”? Circle all that apply.

(i) the columns of $A$ span $\mathbb{R}^n$

(ii) the rows of $A$ span $\mathbb{R}^n$

(iii) the equation $Ax = 0$ has the trivial solution

(iv) the equation $Ax = 0$ has infinitely many solutions

(v) the equation $Ax = b$ is consistent for all $b$ in $\mathbb{R}^n$

(vi) the equation $Ax = b$ has exactly one solution for all $b$ in $\mathbb{R}^n$

(vii) the rank of $A$ is $n$

(viii) the dimension of the null space of $A$ is 0

(ix) $A$ is equal to a product of elementary matrices

(x) $A^5$ is invertible
8. Suppose that \( A \) is a \( 2 \times 2 \) matrix and that \[
Ax = \begin{pmatrix}
7 \\
-3
\end{pmatrix}
\]
has exactly one solution. Is the equation \[
Ax = \begin{pmatrix}
5 \\
9
\end{pmatrix}
\]
consistent? Answer yes/no/maybe and explain.

Suppose that \( A \) is a \( 2 \times 2 \) matrix with two identical columns. Is the equation \[
Ax = \begin{pmatrix}
7 \\
3
\end{pmatrix}
\]
consistent? Answer yes/no/maybe and explain.
9. Determine if the following matrix is invertible and, if so, find the inverse.

\[ A = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 2 & 0 & 11 \end{pmatrix} \]
10. Suppose that $A$ and $B$ are square matrices and that $B$ is the inverse of $A^2$. Express the inverse of $A$ in terms of $A$ and $B$.

What is the area of the parallelogram with vertices $(-5, 3), (0, 5), (47, 24)$, and $(52, 26)$?

Let $B$ be a ball of radius 1 in $\mathbb{R}^3$. The volume of $B$ is $4\pi/3$. Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 9 \\ 0 & 0 & 2 \end{pmatrix}$$

What is the volume of $T_A(B)$?
11. Find the adjugate of the matrix

\[ A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \]

Remember that the adjugate of a matrix is the matrix whose \(ij\)th entry is the \(ji\)th cofactor of \(A\).

What is \(A^{-1}\)?
12. Suppose that $A$ is a $5 \times 5$ matrix with determinant 3.

Is $A$ invertible?

What is $\det(-A)$?

What is $\det(A^{-1})$?

What is $\det A^T$?

What is the determinant of the matrix obtained from $A$ by replacing the first row with twice the first row plus the second row?