Name ________________________

Section HP ___

Mathematics 1553

Midterm 2

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1. Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Find a basis for the column space of A.

What is the dimension of the column space of A?

2. Consider again the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Find a basis for the null space of A.

What is the dimension of the null space of A?

3. Let

$$b_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ and } b_2 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

and $B = \{b_1, b_2\}$. Explain why B is a basis for $\mathbb{R}^2$.

What is $[e_1]_B$? In other words, what are the B–coordinates of $e_1 = (1, 0)$?

4. Consider the set of vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

with $xyz = 0$. Do these vectors form a subspace of $\mathbb{R}^3$? Explain your answer.

Suppose that A is a 5×2 matrix and that the image of the associated linear transformation is a 2–dimensional plane. Describe the set of solutions to $Ax = 0$. Explain your answer.

5. Find an LU factorization of the following matrix:

$$A = \begin{pmatrix} 5 & -1 & 2 & 3 \\ 10 & 3 & 7 & 6 \\ 0 & 20 & 13 & 5 \end{pmatrix}$$

6. The matrix

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$$

has the LU factorization $A = LU$ where

$$L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}.$$

Use this factorization to solve

$$Ax = \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$$

Show clearly the two steps.

7. Let A be an $n \times n$ matrix. Which of the following statements are equivalent to the statement "A is invertible"? Circle all that apply.

(i) the columns of A span $\mathbb{R}^n$

(ii) the rows of A span $\mathbb{R}^n$

(iii) the equation $Ax = 0$ has the trivial solution

(iv) the equation $Ax = 0$ has infinitely many solutions

(v) the equation $Ax = b$ is consistent for all b in $\mathbb{R}^n$

(vi) the equation $Ax = b$ has exactly one solution for all b in $\mathbb{R}^n$

(vii) the rank of A is n

(viii) the dimension of the null space of A is 0

(ix) A is equal to a product of elementary matrices

(x) A^5 is invertible

8. Suppose that A is a 2×2 matrix and that

$$Ax = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

has exactly one solution. Is the equation

$$Ax = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

consistent? Answer *yes/no/maybe* and explain.

Suppose that A is a 2×2 matrix with two identical columns. Is the equation

$$Ax = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

consistent? Answer *yes/no/maybe* and explain.

9. Determine if the following matrix is invertible and, if so, find the inverse.

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 2 & 0 & 11 \end{pmatrix}$$

10. Suppose that A and B are square matrices and that B is the inverse of A^2. Express the inverse of A in terms of A and B.

What is the area of the parallelogram with vertices $(-5, 3)$, $(0, 5)$, $(47, 24)$, and $(52, 26)$?

Let B be a ball of radius 1 in $\mathbb{R}^3$. The volume of B is $4\pi/3$. Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 9 \\ 0 & 0 & 2 \end{pmatrix}$$

What is the volume of $T_A(B)$?

11. Find the adjugate of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Remember that the adjugate of a matrix is the matrix whose ijth entry is the jith cofactor of A.

What is A^{-1}?

12. Suppose that A is a 5×5 matrix with determinant 3.

Is A invertible?

What is $\det(-A)$?

What is $\det(A^{-1})$?

What is $\det A^T$?

What is the determinant of the matrix obtained from A by replacing the first row with twice the first row plus the second row?