Name ________________________

Section _____

Mathematics 1553

Midterm 3

Prof. Margalit

1. Define *eigenvector*.

Define *diagonalizable*.

2. Is $(1, -1, -1)$ an eigenvector of

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{pmatrix}?$$

If so, find the eigenvalue.

Find a basis for the eigenspace with eigenvalue 2 for the matrix

$$A = \begin{pmatrix} 3 & -3 & 3 \\ -2 & 8 & -6 \\ -1 & 3 & -1 \end{pmatrix}.$$

3. Suppose that A is a 2×2 matrix and the associated linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ is orthogonal projection onto the y-axis. List the eigenvalues of A (if there are any) and give a basis for each corresponding eigenspace.

Suppose that A is a 2×2 matrix and that the associated linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ is rotation about the origin by $\pi/4$. List the eigenvalues of A (if there are any) and give a basis for each corresponding eigenspace.

4. Suppose that A is a 6×6 matrix and that it has an eigenvalue λ with algebraic multiplicity 3. What are the possible dimensions for the corresponding eigenspace?

Suppose that A is a 6×6 matrix and that it has an eigenvalue λ with algebraic multiplicity 6. Is A diagonalizable? Answer *yes / no / maybe.*

Suppose that A is a 6×6 matrix and that it has an eigenvalues 1, 2, 3, 4, 5, and 6. Is A diagonalizable? Answer *yes / no / maybe.*

5. Answer *yes/no/maybe* for each question.

Suppose A is a 2×2 matrix that is row equivalent to the identity. Is A diagonalizable?

Suppose A is a 2×2 matrix with two distinct eigenvalues. Is A invertible?

Suppose A is a 2×2 matrix with only one eigenvalue, which is 1. Is A diagonalizable?

Suppose A is an $n \times n$ matrix that is similar to the identity matrix. Is A diagonalizable?

Suppose A is a 5×5 matrix with eigenvalues 1, 2, 3, and 4 and the dimension of the eigenspace for the eigenvalue 3 is 2. Is A diagonalizable?

6. Find the characteristic polynomial for the following matrix:

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -1 & 1 & 1 \\ 2 & 2 & -5 \end{pmatrix}$$

What are the eigenvalues of A?

7. Consider the following matrix:

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

Is A diagonalizable? If so, diagonalize it. If not, explain why not.

8. Consider the following matrix:

$$A = \begin{pmatrix} 7 & -6 \\ 1 & 2 \end{pmatrix}$$

which satisfies

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}^{-1}$$

Give a formula for A^{100}. Your answer should be a single matrix, but the entries do not need to be simplified.

Which describes the linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^2$?

(a) It stretches $(3, 1)$ by 4 and $(2, 1)$ by 5.

(b) It stretches $(1, -1)$ by 4 and $(-2, 3)$ by 5.

(c) It stretches $(3, 1)$ by 5 and $(2, 1)$ by 4.

(d) It stretches $(1, -1)$ by 5 and $(-2, 3)$ by 4.

What is the limit of the slope of $A^k(e_1)$ as k tends to infinity?

(a) $1/3$

(b) 2

(c) $1/2$

(d) $-1/2$

9. Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & -3 \end{pmatrix}$$

Is A diagonalizable? If so, diagonalize it. If not, explain why it is not diagonalizable.

Ask your friend (or enemy) to make up a diagonalization problem for you. Here's how. They choose a matrix B that is either diagonal or not. Then they choose an invertible matrix C and tell you the product $A = CBC^{-1}$ (without telling you what B and C are). Then you figure out if A is diagonalizable or not.

10. Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$:

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

11. Suppose $A^T A = I$. What are the possible values for the determinant of A?

Suppose that A is a matrix where the entries in each column add up to 1 (for example, the matrices in the Google Pagerank algorithm). Show that A has an eigenvalue equal to 1. *Hint: find an eigenvector for* A^T.

12. A rental car agency has two locations. One fourth of the cars from the first location get returned to the first location and three-fourths to the second. Two-thirds of the cars from the second location get returned to the first location and one-third to the second. How should the agency distribute their cars in order to minimize the number of cars that need to be shuttled?

Explain why similar matrices have the same eigenvalues.