Name ________________________

Section ____

Mathematics 1553

Sample Final Exam

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1. Define *eigenvector*.

Suppose that $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation that is orthogonal projection onto some line in $\mathbb{R}^2$ and that A is the associated standard matrix. What are all of the eigenvalues of A?

2. Answer *yes/no/maybe* for each question and explain.

Suppose that 0 is an eigenvalue of A. Is A invertible?

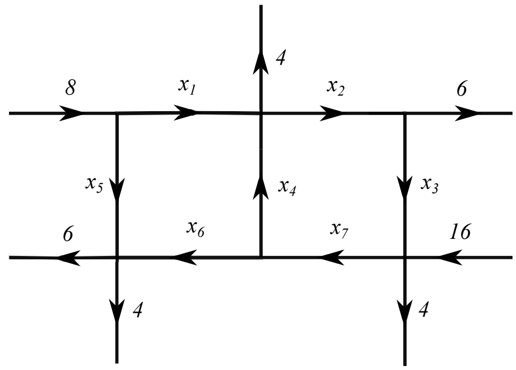
Suppose that A is a 2×2 matrix with two identical columns. Is the equation

$$Ax = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

consistent?

Suppose A is a 5×5 matrix with eigenvalues 1, 2, 3, and 4 and the algebraic multiplicity of the eigenvalue 3 is 2. Is A diagonalizable?

3. The following diagram indicates traffic flow in one part of town:



Write a system of linear equations in x_1, x_2, x_3, x_4, x_5, x_6, and x_7 describing the traffic flow around the two squares.

If you row reduce the augmented matrix corresponding to your system of linear equations, you will get the matrix:

$$A = \left(\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & -6 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -12 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Find a parametric solution for the possible traffic flows around the two squares.

In your parametric solution on the previous page, which values of the parameter are allowable, assuming traffic must flow in the direction of the arrow?

What if the streets labeled with x_1 and x_2 need to be closed? How can traffic be re-routed? In your answer, explain which streets would have to have their directions reversed, and give the amount of traffic flow on each street.

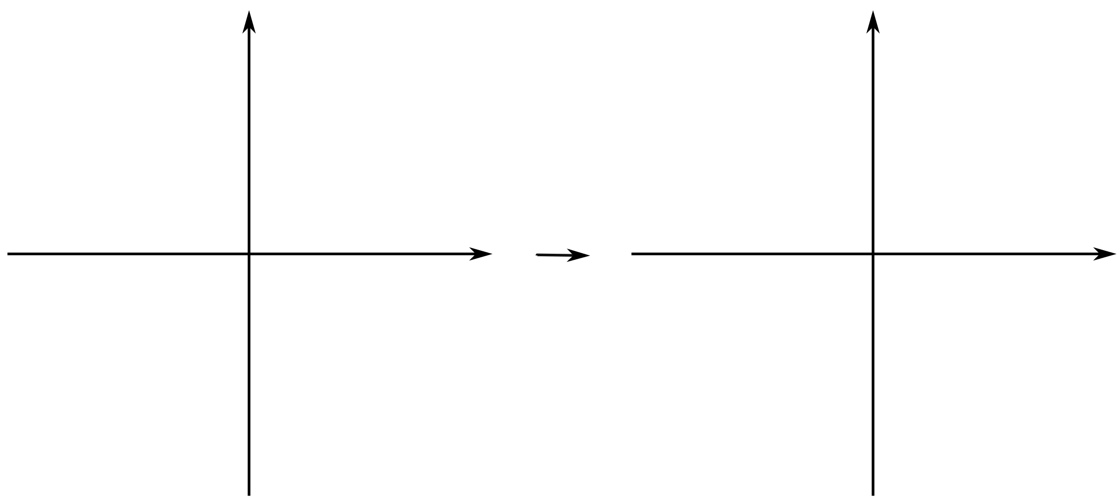
4. Consider the matrix

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Let T_A be the associated linear transformation.

Is T_A one-to-one? onto?

On the right side draw the range of T_A. On the left side draw and label the null space of A. Then choose any nonzero point b in the range of T_A and label it with its (x, y)–coordinates. On the left side draw and label the set of points x in the domain with $T_A(x) = b$.



On the left you should have drawn two lines $\mathbb{R}^2$. What geometric relation do they satisfy?

(a) they are orthogonal

(b) they intersect at the origin

(c) they are parallel

(d) they are equal

5. Find an LU decomposition of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 6 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

Use your LU decomposition from the previous page to solve the equation $Ax = b$ where

$$b = \begin{pmatrix} 2 \\ 8 \\ 0 \end{pmatrix}$$

Show clearly the two steps.

6. Determine whether or not the following matrix is invertible:

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{pmatrix}$$

If it is invertible, find the inverse. If it is not, explain why not.

7. Find the eigenvalues of the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

Hint: Look for common factors before multiplying everything out!

Is A (from the previous page) diagonalizable? If so, diagonalize it. If not, explain why not.

8. Perform the Gram–Schmidt process on the following set of vectors:

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ -1 \end{pmatrix} \right\}$$

Find the projection of the vector $e_1 = (1, 0, 0, 0)$ to the subspace of $\mathbb{R}^4$ spanned by the three vectors at the top of the page.

9. Find a QR factorization of the following matrix:

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix}$$

10. Find a least squares solution $\hat{x}$ to the equation $Ax = b$ where

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

What best describes the least squares solution $\hat{x}$?

(a) $A\hat{x}$ is the closest point in $\mathbb{R}^3$ to the column space of A

(b) $A\hat{x}$ is the closest point in the column space of A to the vector $(1, 4, 2)$

(c) $\hat{x}$ is the closest point in $\mathbb{R}^3$ to the column space of A

(d) $\hat{x}$ is the closest point in the column space of A to the vector $(1, 4, 2)$

Problem	Score
1	
2	
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