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Mathematics 1553

Quiz 2

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Section J: left / center / right

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1. Say that u and v are vectors and neither is a multiple of the other. Then Span$\{u, v\}$ is...

(a) a line through the origin

(b) the line through the origin and u plus the line through the origin and v

(c) a plane through the origin $\vec{0} = 0 \cdot \vec{u} + 0 \cdot \vec{v}$

(d) a plane, but not necessarily through the origin

Is the vector $\begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$ in the span of the columns of $\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix}$? If so, write it as a linear combination of the columns.

If $\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix} \vec{x} = \begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$ is consistant, then $\begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$ is in the span.

$$\left(\begin{array}{cc|c} 1 & 1 & 6 \\ 1 & 0 & 5 \\ 1 & 2 & 7 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_1 - R_2} \left(\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow x = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\boxed{\text{Yes}}$, $\begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$ is in the span

$$\Rightarrow \begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

Turn the page over!

Do the columns of the following matrix span $\mathbb{R}^3$? Why or why not?

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & 2 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So A does not have 3 pivots, and A needs 3 pivots to have its columns span $\mathbb{R}^3$.

$\Rightarrow$ Columns of A does not span $\mathbb{R}^3$.