Name SOLUTIONS.

Section H
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Mathematics 1553

Quiz 4
Prof. Margalit
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1. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \to (-1)R_3}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right]$$

A^{-1}

Note that you cannot do a step like $R_3 \to (-1)R_3 + 2R_1$ without changing the sign of R_3 on the identity matrix as well!

A^{-1}

Use your answer from the previous question to solve the matrix equation

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

We know $A\vec{x} = \vec{y} \Rightarrow \vec{x} = A^{-1}\vec{y}$

$$\Rightarrow \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

Note: If you did not use A^{-1}, then points were not awarded

$$= \begin{bmatrix} -1 \times 2 + 0 \times 2 + 1 \times 3 \\ -2 \times 2 + 1 \times 2 + 1 \times 3 \\ 2 \times 2 + 0 \times 2 - 1 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 3 \\ -4 + 2 + 3 \\ 4 - 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Turn the page over!

Suppose that A, B, C, $A+B$, and X are invertible $n \times n$ matrices. Solve for X:

$$X(A+B) + B = C$$

$$X(A+B) = C - B$$

$$X(A+B)(A+B)^{-1} = (C-B)(A+B)^{-1}$$

$$X \, 1 = (C-B)(A+B)^{-1}$$

$$\boxed{X = (C-B)(A+B)^{-1}}$$