Name Solution

Section J

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Mathematics 1553

Quiz 5

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26 February 2016

1. Suppose that A is an $n \times n$ matrix and that $Ax = b$ is consistent for all b in $\mathbb{R}^n$. How many solutions can $Ax = 0$ have? Circle all that apply.

(a) no solutions

(b) one solution (A is invertible)

(c) two solutions

(d) infinitely many solutions

2. Find an LU decomposition of the matrix

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \\ 0 & 8 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 3 \\ 0 & 8 \end{pmatrix} \xrightarrow{-R_1+R_2 \to R_2} \begin{pmatrix} 1 & -1 \\ 0 & 4 \\ 0 & 8 \end{pmatrix} \xrightarrow{-2R_2+R_3 \to R_3} \underbrace{\begin{pmatrix} 1 & -1 \\ 0 & 4 \\ 0 & 0 \end{pmatrix}}_{U}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 4 \\ 0 & 0 \end{pmatrix}$$

Turn the page over!

3. Use the LU decomposition

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -2 \end{pmatrix}$$

to solve $Ax = b$ where

$$b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Show clearly the two steps.

$L\vec{y} = \vec{b}$

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 2 & 1 & 0 \end{array}\right)$$

$y_1 = 1$

$2y_1 + y_2 = 0$

$\Rightarrow \begin{cases} y_1 = 1 \\ y_2 = -2 \end{cases}$

$\vec{y} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$U\vec{x} = \vec{y}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & -2 & -2 \end{array}\right)$$

$\Downarrow$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{array}\right)$$

$x_1 + x_3 = 1$

$x_2 + 2x_3 = 2$

$\Downarrow$

$x_1 = 1 - x_3$

$x_2 = 2 - 2x_3$

$x_3 = x_3$ (free)

$$\boxed{\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} x_3}$$