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Mathematics 1553

Quiz 6
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4 March 2016

1. Find the rank and the dimension of the null space of the following matrix:

$$A = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 2 \end{pmatrix}}_{5} \rightsquigarrow \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} \boxed{1} & 0 & 1 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Rank $(A) = 2$

dim Null $(A) = 5 - 2 = 3$

Can you make a 3×5 matrix with rank 3 and the dimension of the null space equal to 3? If so, give an example. If not, explain why not.

There is <u>no</u> such example.

According to the rank theorem, for a $n \times m$ matrix:

rank(A) + dim Null $(A) = m$.

In this case, with rank $(A) = 3$, $m = 5$, dim Null $(A) = 2$.

Turn the page over!

Therefore, the null space can only have dimension equal to 2

2. Consider the following vectors in $\mathbb{R}^2$:

$$b_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad u = \begin{pmatrix} 7 \\ 10 \end{pmatrix}$$

The set $B = \{b_1, b_2\}$ is a basis for $\mathbb{R}^2$. Find $[u]_B$, the B–coordinates of u.

assume $[u]_B = \begin{bmatrix} k \\ j \end{bmatrix}$

Then, $k b_1 + j b_2 = u$.

$\Rightarrow \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} k \\ j \end{bmatrix} = u$

$$\left[\begin{array}{cc|c} 2 & 1 & 7 \\ 3 & 1 & 10 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 1 & 7 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 1 & 7 \\ 0 & 1 & 1 \end{array}\right]$$

$\Rightarrow \begin{cases} k = 3 \\ j = 1 \end{cases}$

$\Rightarrow [u]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$