Name Solution

Section H
Subsection left center right

Mathematics 1553

Quiz 7
Prof. Margalit
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1. Assume that A is an $n \times n$ matrix, v is a vector in $\mathbb{R}^n$ and λ is a real number. Which of the following correctly characterizes v and λ as an eigenvector and eigenvalue for A?

(a) $Av = \lambda v$
(b) $Av = \lambda v$ and $v \neq 0$
(c) $Av = \lambda v$ and $\lambda \neq 0$
(d) $Av = \lambda v$, $v \neq 0$, and $\lambda \neq 0$
(e) none of the above

Consider the following matrix

$$A = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$$

Determine whether or not each of the following vectors is an eigenvector for A. If so, give the corresponding eigenvalue.

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ YES NO EIGENVALUE: -1

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ YES NO EIGENVALUE:

$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ YES NO EIGENVALUE:

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ YES NO EIGENVALUE: 1

Turn the page over!

For the matrix below, find a basis for the eigenspace corresponding to the eigenvalue -1.

$$\begin{pmatrix} -3 & 0 & 2 \\ -4 & -1 & 4 \\ -2 & 0 & -1 \end{pmatrix}$$

$(A - I\lambda)\vec{v} = 0$

$$\begin{bmatrix} -3+1 & 0 & 2 \\ -4 & -1+1 & 4 \\ -2 & 0 & -1+1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -2 & 0 & 2 & 0 \\ -4 & 0 & 4 & 0 \\ -2 & 0 & 0 & 0 \end{array}\right]$$

$\Downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_1 = 0$

$\Rightarrow v_2 = \text{free}. \ (v_2 = v_2)$

$v_3 = 0$

$\Rightarrow \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v_2$

$\boxed{\text{basis } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$