Mathematics 1553
Quiz 8
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1. Consider the following matrix

$$A = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}$$

Find the characteristic polynomial of A.

$$\det(A - I\lambda) = \begin{vmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = (6-\lambda)(3-x) + 2$$

Find the eigenvalues of A.

$$(6-\lambda)(3-\lambda) + 2 = 0$$

$$\lambda^2 - 9\lambda + 20 = 0$$

$$(\lambda - 5)(\lambda - 4) = 0$$

$$\lambda = 5$$

$$\lambda = 4$$

Turn the page over!

For each eigenvalue of A, find one eigenvector.

$$\lambda = 5 \quad \begin{pmatrix} 6-5 & -1 & | & 0 \\ 2 & 3-5 & | & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 = x_2 \Rightarrow \vec{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda = 4 \quad \begin{pmatrix} 6-4 & -1 & | & 0 \\ 2 & 3-4 & | & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -\frac{1}{2} & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

$$2x_1 = x_2$$

$$\vec{v_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Find a diagonalization of A.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^{-1}$$