Name Solution

Section J
Subsection left center right

Mathematics 1553

Quiz 9
Prof. Margalit
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1. Consider the following subspace of $\mathbb{R}^4$:

$$W = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}.$$

Determine the orthogonal projection to W of the following vector:

$$v = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

① check: $w_1 \cdot w_2 = (1,1,1,1)\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = 0$

Therefore the formula can be applied.

$w_1 \cdot v = 2$

$w_1 \cdot w_1 = 4$

$w_2 \cdot v = 0$

$w_2 \cdot w_2 = 4$

$$\Rightarrow \quad \text{proj}_{\langle w \rangle}(v) = \frac{2}{4} w_1 + \frac{0}{4} w_2$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Find a basis for $W^\perp$.

Form matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{pmatrix}$

$$\boxed{\text{A basis of } W^\perp \text{ is } \left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}}$$

The basis of $W^\perp$ is the null space of A^T, since every vector in $W^\perp$ will be perpendicular to w_1 and w_2.

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \Rightarrow \begin{cases} x_1 = -x_2 - x_3 - x_4 \\ x_2 = x_2 \text{ (free)} \\ x_3 = -x_4 \\ x_4 = x_4 \text{ (free)} \end{cases} \Rightarrow x_2\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$