Math 1553: Introduction to Linear Algebra

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Linear. Algebra.

What is Linear Algebra?

Linear

Algebra

- from al-jebr (Arabic), meaning reunion of broken parts
- 9^{th} century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

Why a whole course?

Engineers need to solve *lots* of equations in *lots* of variables.

$$\begin{aligned}
3x_1 + 4x_2 + 10x_3 + 19x_4 - 2x_5 - 3x_6 &= 141 \\
7x_1 + 2x_2 - 13x_3 - 7x_4 + 21x_5 + 8x_6 &= 2567 \\
-x_1 + 9x_2 + \tfrac{3}{2}x_3 + x_4 + 14x_5 + 27x_6 &= 26 \\
\tfrac{1}{2}x_1 + 4x_2 + 10x_3 + 11x_4 + 2x_5 + x_6 &= -15
\end{aligned}$$

Often, it's enough to know some information about the set of solutions without having to solve the equations at all!

In real life, the difficult part is often in recognizing that a problem can be solved using linear algebra in the first place: need *conceptual* understanding.

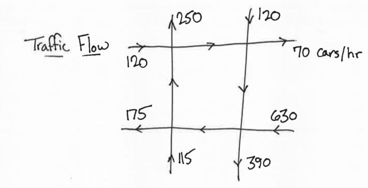
Linear Algebra in Engineering

Almost every engineering problem, no matter how huge, can be reduced to linear algebra:

$$
\begin{aligned}
Ax &= b \qquad \text{or} \\
Ax &= \lambda x \qquad \text{or} \\
Ax &\approx x
\end{aligned}
$$

Applications of Linear Algebra

Civil Engineering: How much traffic lies in the four unlabeled segments?



Applications of Linear Algebra

Chemistry: Balancing reaction equations

$$\underline{\ \ x\ \ }\ C_2H_6 + \underline{\ y\ }\ O_2 \rightarrow \underline{\ \ z\ \ }\ CO_2 + \underline{\ \ w\ \ }\ H_2O$$

Applications of Linear Algebra

Biology: In a population of rabbits...

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population in 2016 (in terms of the number of first, second, and third year rabbits), then what is the population in 2017?

Say the numbers of first, second, and third year rabbits in year n are:

$$F_n, S_n, T_n$$

Applications of Linear Algebra

Geometry and Astronomy: Find the equation of a circle passing through 3 given points, say (1,0), (0,1), and (1,1). The general form of a circle is $a(x^2 + y^2) + bx + cy + d = 0 \rightsquigarrow$ system of linear equations.

Very similar to: compute the orbit of a planet: $a(x^2 + y^2) + bx + cy + d = 0$

Applications of Linear Algebra

Google: “The 25 billion dollar eigenvector.” Each web page has some importance, which it shares via outgoing links to other pages $\rightsquigarrow$ system of linear equations. Stay tuned!

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  - Find best-fit solutions to systems of linear equations that have no actual solution using least squares approximations