

Name \_\_\_\_\_

# Mathematics 1553

Midterm 1

Prof. Margalit

Section    D1/Isabella    D2/Kyle    D3/Kalen    D4/Sidhanth    (circle one!)

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1. Answer the following questions. No justification for your answer is required.

Is the matrix  $\left( \begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right)$  in reduced row echelon form?

YES

NO

Is the vector  $\begin{pmatrix} 99 \\ 97 \end{pmatrix}$  a linear combination of the vectors  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ 6 \end{pmatrix}$ ?

YES

NO

Suppose  $A$  is a  $2 \times 2$  matrix and  $A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \end{pmatrix}$ . Is it possible that the set of solutions to  $Ax = 0$  is the line  $x_1 = x_2$ ?

YES

NO

Suppose  $A$  is a  $4 \times 5$  matrix. Is it possible that  $Ax = b$  is consistent for all  $b$  in  $\mathbb{R}^4$ ?

YES

NO

Suppose that  $v_1$ ,  $v_2$ , and  $v_3$  are vectors in  $\mathbb{R}^5$ . Must it be true that  $v_1$ ,  $v_2$ , and  $v_3$  are linearly independent?

YES

NO

EXTRA SPACE FOR WORK ON PAGE 1

2. Answer the following questions. No justification for your answer is required.

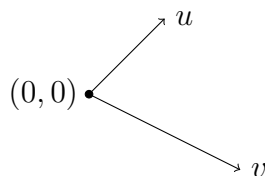
Complete the following definition: *Vectors  $v_1, \dots, v_k$  in  $\mathbb{R}^n$  are linearly independent if...*

Write down one vector in  $\mathbb{R}^3$  that is not in the span of the vectors  $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ .

Find a matrix  $A$  so that the set of solutions to  $Ax = 0$  is a line in  $\mathbb{R}^3$  and so that the equation  $Ax = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$  is consistent.

Circle the formula that best describes  $w$  in terms of  $u$  and  $v$ .

$w \bullet$



$u - v$

$v - u$

$-u - v$

$2u - v$

EXTRA SPACE FOR WORK ON PAGE 2

3. Suppose that  $A$  is a  $5 \times 6$  matrix with 2 pivots, and that  $Ax = b$  is a matrix equation with  $b$  nonzero. Fill in the three blanks and answer the two multiple choice questions.

The set of solutions to  $Ax = b$  is a -dimensional plane in  $\mathbb{R}^{\text{$ .

The vector  $b$  lies in  $\mathbb{R}^{\text{$ .

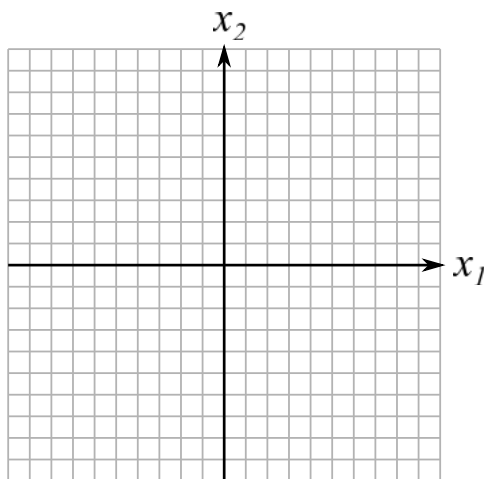
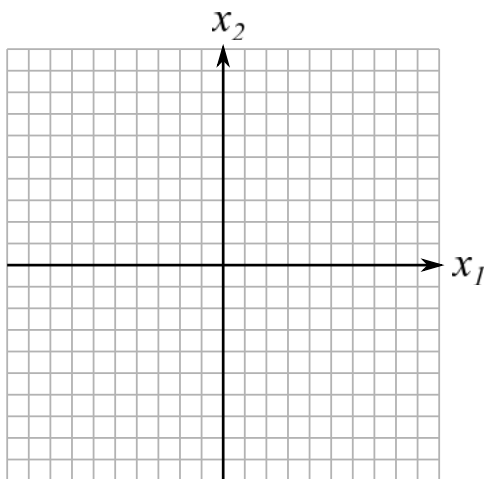
Is the solution set to  $Ax = b$  equal to a span?    YES        NO        MAYBE

Which phrase best describes the relationship between the solutions to  $Ax = 0$  and  $Ax = b$ ?

SAME            PARALLEL            MEET IN ONE POINT

4. Consider the matrix  $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$ . Draw and label the following 5 things.

1. On the *right-hand side* draw the span of the columns of  $A$ .
2. On the *right-hand side*, draw a dot for a non-zero vector  $b$  so  $Ax = b$  is consistent.
3. On the *left-hand side* draw the solutions to  $Ax = b$  for your choice of  $b$ .
4. On the *left-hand side*, draw an arrow for one particular solution to  $Ax = b$ .
5. On the *left-hand side*, draw the solutions to  $Ax = 0$ .



EXTRA SPACE FOR WORK ON PAGE 3



5. Find the reduced row echelon form of the following matrix. Show your work.

$$\begin{pmatrix} 0 & 0 & 1 & 2 \\ 1 & 3 & -2 & 1 \\ 2 & 6 & 0 & 10 \end{pmatrix}$$

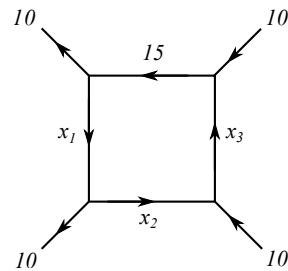
6. Suppose that there is a matrix equation  $Ax = b$  and that the reduced row echelon form of the augmented matrix  $(A|b)$  is

$$\left( \begin{array}{cccc|c} 0 & 1 & -3 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Write the parametric vector form of the solution to  $Ax = b$ .

EXTRA SPACE FOR WORK ON PAGE 4

7. The following diagram indicates traffic flow in the town square (the numbers indicate the number of cars per minute on each section of road).



Write down a **vector equation** describing the flow of traffic. Do not solve.

8. Find all values of  $h$  so that the vectors  $\begin{pmatrix} 1 \\ 1 \\ -9 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$ , and  $\begin{pmatrix} 1 \\ h \\ h \end{pmatrix}$  are linearly dependent. Show your work.

EXTRA SPACE FOR WORK ON PAGE 5

EXTRA SPACE

| Problem | Score |
|---------|-------|
| 1       |       |
| 2       |       |
| 3       |       |
| 4       |       |
| 5       |       |
| Total   |       |