Name SOLUTIONS

Mathematics 1553

Midterm 1

Prof. Margalit

Section D1/Isabella D2/Kyle D3/Kalen D4/Sidhanth (circle one!)

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1. Answer the following questions. No justification for your answer is required.

Is the matrix $\left(\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$ in reduced row echelon form?

YES NO

Is the vector $\binom{99}{97}$ a linear combination of the vectors $\binom{3}{4}$ and $\binom{5}{6}$?

YES NO

Suppose A is a 2×2 matrix and $A\binom{1}{1} = \binom{19}{7}$. Is it possible that the set of solutions to $Ax = 0$ is the line $x_1 = x_2$?

YES NO

Suppose A is a 4×5 matrix. Is it possible that $Ax = b$ is consistent for all b in $\mathbb{R}^4$?

YES NO

Suppose that v_1, v_2, and v_3 are vectors in $\mathbb{R}^5$. Must it be true that v_1, v_2, and v_3 are linearly independent?

YES NO

2. Answer the following questions. No justification for your answer is required.

Complete the following definition: *Vectors $v_1, \ldots, v_k$ in $\mathbb{R}^n$ are linearly independent if...*

the only soln to vec. eqn

$$x_1 v_1 + \cdots + x_k v_k = 0$$

is trivial.

Write down one vector in $\mathbb{R}^3$ that is not in the span of the vectors $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

not equal

Find a matrix A so that the set of solutions to $Ax = 0$ is a line in $\mathbb{R}^3$ and so that the equation $Ax = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ is consistent.

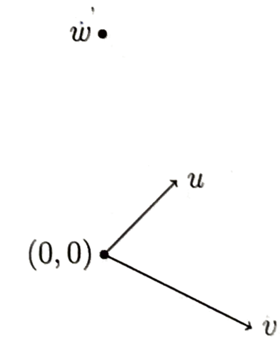
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4×3

2 pivots

e_3 in span of cols

Circle the formula that best describes w in terms of u and v.



$u - v$ $\qquad$ $v - u$ $\qquad$ $-u - v$ $\qquad$ $2u - v$

3. Suppose that A is a 5×6 matrix with 2 pivots, and that $Ax = b$ is a matrix equation with b nonzero. Fill in the three blanks and answer the two multiple choice questions.

The set of solutions to $Ax = b$ is a $\boxed{4}$-dimensional plane in $\mathbb{R}^{\boxed{6}}$.

The vector b lies in $\mathbb{R}^{\boxed{5}}$.

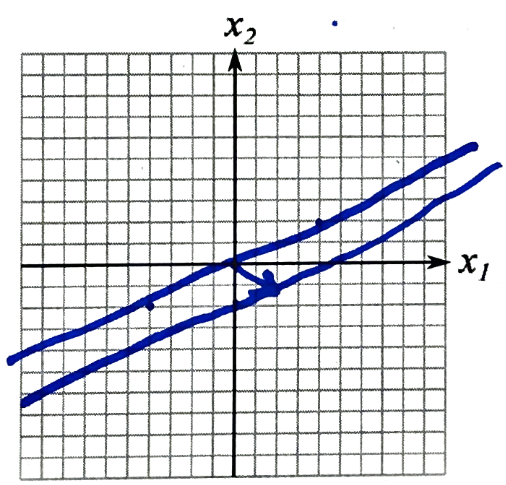
Is the solution set to $Ax = b$ equal to a span? YES NO MAYBE

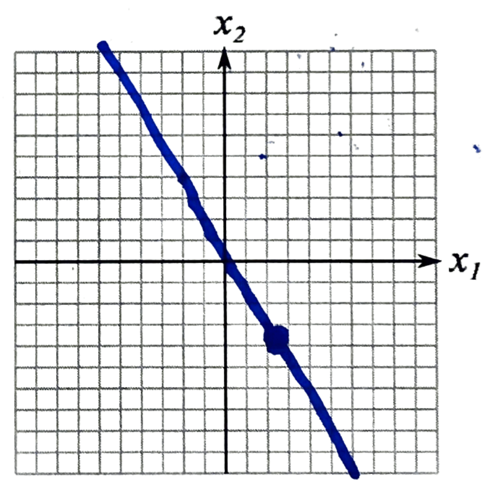
Which phrase best describes the relationship between the solutions to $Ax = 0$ and $Ax = b$?

SAME PARALLEL MEET IN ONE POINT

4. Consider the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$. Draw and label the following 5 things.

1. On the *right-hand side* draw the span of the columns of A.
2. On the *right-hand side*, draw a dot for a non-zero vector b so $Ax = b$ is consistent.
3. On the *left-hand side* draw the solutions to $Ax = b$ for your choice of b.
4. On the *left-hand side*, draw an arrow for one particular solution to $Ax = b$.
5. On the *left-hand side*, draw the solutions to $Ax = 0$.





5. Find the reduced row echelon form of the following matrix. Show your work.

$$\begin{pmatrix} 0 & 0 & 1 & 2 \\ 1 & 3 & -2 & 1 \\ 2 & 6 & 0 & 10 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & -2 & 1 \\ 2 & 6 & 0 & 10 \\ 0 & 0 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -2 & 1 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & -2 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

6. Suppose that there is a matrix equaion $Ax = b$ and that the reduced row echelon form of the augmented matrix $(A|b)$ is

$$\left(\begin{array}{cccc|c} 0 & 1 & -3 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Write the parametric vector form of the solution to $Ax = b$.

$x_1 = x_1$

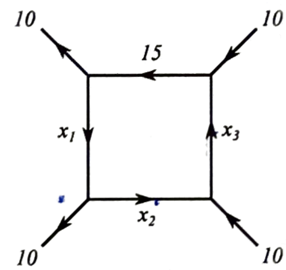
$x_2 = 3x_3 + 7$

$x_3 = x_3$

$x_4 = 2$

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \\ 0 \\ 2 \end{pmatrix}$$

7. The following diagram indicates traffic flow in the town square (the numbers indicate the number of cars per minute on each section of road).



Write down a **vector equation** describing the flow of traffic. Do not solve.

$$\begin{aligned} 10 + x_1 &= 15 \\ x_1 &= 10 + x_2 \\ 10 + x_2 &= x_3 \\ 10 + x_3 &= 15 \end{aligned} \rightsquigarrow \begin{aligned} x_1 &= 5 \\ x_1 + x_2 &= 10 \\ -x_2 + x_3 &= 10 \\ x_3 &= 5 \end{aligned} \rightsquigarrow x_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 10 \\ 5 \end{pmatrix}$$

8. Find all values of h so that the vectors $\begin{pmatrix} 1 \\ 1 \\ -9 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ h \\ h \end{pmatrix}$ are linearly dependent. Show your work.

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & h \\ -9 & 6 & h \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & h-1 \\ 0 & 6 & h+9 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & h-1 \\ 0 & 0 & -5h+15 \end{pmatrix}$$

$$\boxed{h = 3}$$