1. Answer the following questions. No justification for your answer is required.

Is the matrix \( \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \) in reduced row echelon form?

YES \hspace{1cm} NO

Is the vector \( \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 7 \end{pmatrix} \) a linear combination of the vectors \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 5 \end{pmatrix} \)?

YES \hspace{1cm} NO

Suppose \( A \) is a \( 2 \times 2 \) matrix and \( A \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \end{pmatrix} \). Is it possible that the set of solutions to \( Ax = 0 \) is the line \( x_1 = x_2 \)?

YES \hspace{1cm} NO

Suppose \( A \) is a \( 4 \times 5 \) matrix. Is it possible that \( Ax = b \) is consistent for all \( b \) in \( \mathbb{R}^4 \)?

YES \hspace{1cm} NO

Suppose that \( v_1, v_2, \) and \( v_3 \) are vectors in \( \mathbb{R}^5 \). Must it be true that \( v_1, v_2, \) and \( v_3 \) are linearly independent?

YES \hspace{1cm} NO
2. Answer the following questions. No justification for your answer is required.

Complete the following definition: *Vectors* \( v_1, \ldots, v_k \) *in* \( \mathbb{R}^n \) *are linearly independent if...*

\[
\text{the only soln to vec. eqn } \sum_{i=1}^{k} x_i v_i = 0 \text{ is trivial.}
\]

Write down one vector in \( \mathbb{R}^3 \) that is not in the span of the vectors \( \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \).

\[
\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \nleftrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{not equal}
\]

Find a matrix \( A \) so that the set of solutions to \( Ax = 0 \) is a line in \( \mathbb{R}^3 \) and so that the equation \( Ax = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \) is consistent.

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

4 \times 3, 2 \text{ pivots, } e_3 \text{ in span of cols}

Circle the formula that best describes \( w \) in terms of \( u \) and \( v \).

\[
w = \ldots
\]

\[
\begin{pmatrix} 0, 0 \end{pmatrix}
\]

\[
\begin{align*}
u - v & \quad v - u & \quad -u - v \quad 2u - v
\end{align*}
\]
3. Suppose that $A$ is a $5 \times 6$ matrix with 2 pivots, and that $Ax = b$ is a matrix equation with $b$ nonzero. Fill in the three blanks and answer the two multiple choice questions.

The set of solutions to $Ax = b$ is a $\boxed{4}$-dimensional plane in $\mathbb{R}^{\boxed{6}}$.

The vector $b$ lies in $\mathbb{R}^{\boxed{5}}$.

Is the solution set to $Ax = b$ equal to a span? **YES** **NO** **MAYBE**

Which phrase best describes the relationship between the solutions to $Ax = 0$ and $Ax = b$? **SAME** **PARALLEL** **MEET IN ONE POINT**

4. Consider the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$. Draw and label the following 5 things.

1. On the right-hand side draw the span of the columns of $A$.
2. On the right-hand side, draw a dot for a non-zero vector $b$ so $Ax = b$ is consistent.
3. On the left-hand side draw the solutions to $Ax = b$ for your choice of $b$.
4. On the left-hand side, draw an arrow for one particular solution to $Ax = b$.
5. On the left-hand side, draw the solutions to $Ax = 0$.
5. Find the reduced row echelon form of the following matrix. Show your work.

\[
\begin{pmatrix}
0 & 0 & 1 & 2 \\
1 & 3 & -2 & 1 \\
2 & 6 & 0 & 10 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 3 & -2 & 1 \\
2 & 6 & 0 & 10 \\
0 & 0 & 1 & 2 \\
\end{pmatrix} \rightarrow 
\begin{pmatrix}
1 & 3 & -2 & 1 \\
0 & 0 & 4 & 8 \\
0 & 0 & 1 & 2 \\
\end{pmatrix} \rightarrow 
\begin{pmatrix}
1 & 3 & 0 & 5 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

6. Suppose that there is a matrix equation \( Ax = b \) and that the reduced row echelon form of the augmented matrix \((A|b)\) is

\[
\begin{pmatrix}
0 & 1 & -3 & 0 & 7 \\
0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Write the parametric vector form of the solution to \( Ax = b \).

\[
\begin{align*}
    x_1 &= x_1 \\
    x_2 &= 3x_3 + 7 \\
    x_3 &= x_3 \\
    x_4 &= 2
\end{align*}
\]

\[
x_1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 7 \end{pmatrix}
\]
7. The following diagram indicates traffic flow in the town square (the numbers indicate the number of cars per minute on each section of road).

\[
\begin{array}{c}
10 + x_1 = 15 \\
x_1 = 10 + x_2 \\
x_1 + x_2 = 10 \\
10 + x_2 = x_3 \\
-2x_2 + x_3 = 10 \\
10 + x_3 = 15 \\
\end{array}
\]

Write down a vector equation describing the flow of traffic. Do not solve.

\[
10 + x_1 = 15 \quad x_1 = 5 \quad x_1 \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}
\]

8. Find all values of \( h \) so that the vectors \( \begin{pmatrix} 1 \\ 0 \\ h \end{pmatrix} \), \( \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \), and \( \begin{pmatrix} 1 \\ h \end{pmatrix} \) are linearly dependent. Show your work.

\[
\begin{pmatrix} 10 & 1 \\ 1 & h \\ -9 & 6 \end{pmatrix} \sim \begin{pmatrix} 10 & 1 \\ 0 & 1 \ h-1 \\ 0 & 6 \ h+9 \end{pmatrix} \sim \begin{pmatrix} 10 & 1 \\ 0 & 1 \ h-1 \\ 0 & 0 \ -5h+15 \end{pmatrix}
\]

\[ h = 3 \]