Name ________________________

Mathematics 1553

Midterm 2

Prof. Margalit

Section D1/Isabella D2/Kyle D3/Kalen D4/Sidhanth (circle one!)

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1. Answer the following questions. No justification for your answer is required.

- Let V be the set of solutions to $x + y + z = 0$ in $\mathbb{R}^3$. Is V a subspace of $\mathbb{R}^3$?

YES NO

- Say that V is a plane in $\mathbb{R}^3$, that v and w are two vectors in V, and that neither v or w is a multiple of the other. Must it be true that $\{v, w\}$ is a basis for V?

YES NO

- If A is an invertible $n \times n$ matrix, then must it be true that the columns of A form a basis for $\mathbb{R}^n$?

YES NO

- Suppose A is a 4×4 matrix and $Ax = e_1$ has infinitely many solutions. Is A invertible?

YES NO

- Suppose that A is a 2×3 matrix and that the column space for A is a plane. Is it possible for $Ax = b$ to have infinitely many solutions?

YES NO

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2. Answer the following questions. No justification for your answer is required.

- Complete the definition: *Vectors* $v_1, \ldots, v_k$ *form a basis for a subspace* V *of* $\mathbb{R}^n$ *if...*

- Let V be the subset of $\mathbb{R}^2$ given by the first quadrant. In other words:

$$V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a \geq 0 \text{ and } b \geq 0 \right\}$$

Which parts of the definition of a subspace are satisfied by V? Circle all that apply.

(a) the zero vector is in V

(b) if v and w are in V then $v + w$ is in V

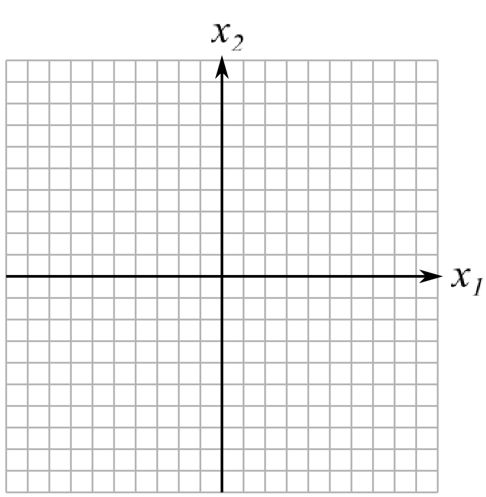
(c) if v is in V and c is a real number then cv is in V

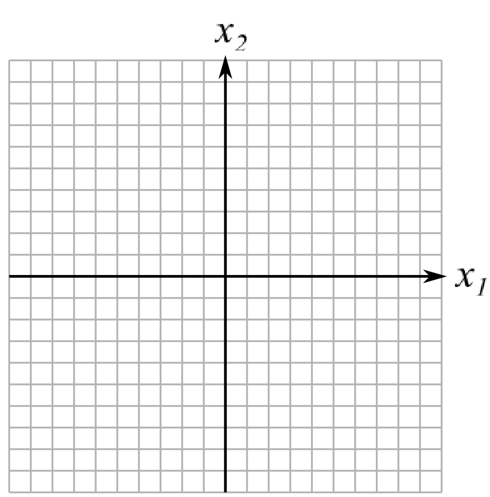
(d) none of the above

- Solve the following equation for X. Assume that all matrices that arise are invertible.

$$X + AX = B$$

- Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. On the *left-hand side*, draw the null space of A. On the *right-hand side* draw the column space of A.





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3. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation given by clockwise rotation by $\pi/4$ and let $U : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by the formula

$$U \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ x - y \end{pmatrix}$$

What is the standard matrix for T?

What is the standard matrix for U?

What is the range of U?

Which of the following two compositions makes sense? $T \circ U$ $U \circ T$

Write down the standard matrix for the composition you chose.

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4. Consider the following matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} 3 & 6 & -1 \\ 4 & 8 & -1 \\ 4 & 8 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Let T be the matrix transformation $T(v) = Av$.

Find a basis for the null space of A.

Find a basis for column space of A.

Are there two different vectors v and w with $T(v) = T(w)$ and $T(v) \neq 0$? YES NO

If you answered yes, find such a v and w. If you answered no, explain why not.

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5. Find the inverse of the following matrix.

$$\begin{pmatrix} 3 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 4 \end{pmatrix}$$

Find a matrix A so the domain of $T(v) = Av$ is $\mathbb{R}^3$ and the range is the line $y = 2x$ in $\mathbb{R}^2$.

Find a 2×2 matrix A so that $A \neq I$ and so that $A^4 = I$. *Hint: Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ so that $T \circ T \circ T \circ T$ is the identity.*

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Problem	Score
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