Name SOLUTIONS

Mathematics 1553

Midterm 2

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Section D1/Isabella D2/Kyle D3/Kalen D4/Sidhanth (circle one!)

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1. Answer the following questions. No justification for your answer is required.

- Let V be the set of solutions to $x + y + z = 0$ in $\mathbb{R}^3$. Is V a subspace of $\mathbb{R}^3$?

YES NO

- Say that V is a plane in $\mathbb{R}^3$, that v and w are two vectors in V, and that neither v or w is a multiple of the other. Must it be true that $\{v, w\}$ is a basis for V?

YES NO

- If A is an invertible $n \times n$ matrix, then must it be true that the columns of A form a basis for $\mathbb{R}^n$?

YES NO

- Suppose A is a 4×4 matrix and $Ax = e_1$ has infinitely many solutions. Is A invertible?

YES NO

- Suppose that A is a 2×3 matrix and that the column space for A is a plane. Is it possible for $Ax = b$ to have infinitely many solutions?

YES NO

2. Answer the following questions. No justification for your answer is required.

- Complete the definition: *Vectors $v_1, \ldots, v_k$ form a basis for a subspace V of $\mathbb{R}^n$ if...*

they are lin. ind. and
they span V

- Let V be the subset of $\mathbb{R}^2$ given by the first quadrant. In other words:

$$V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a \geq 0 \text{ and } b \geq 0 \right\}$$

Which parts of the definition of a subspace are satisfied by V? Circle all that apply.

(a) the zero vector is in V

(b) if v and w are in V then $v + w$ is in V

(c) if v is in V and c is a real number then cv is in V

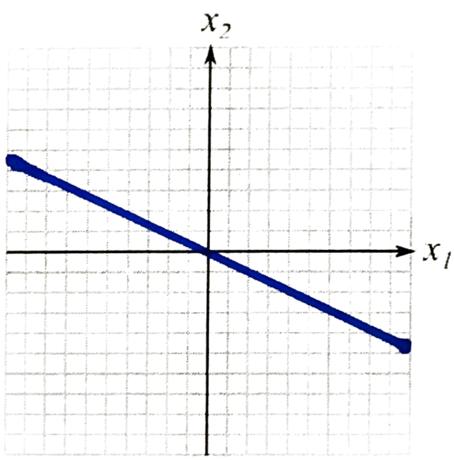
(d) none of the above

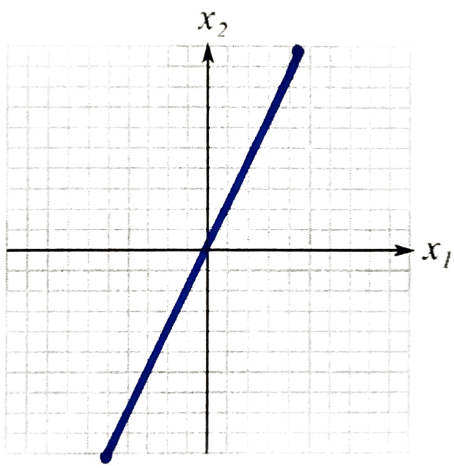
- Solve the following equation for X. Assume that all matrices that arise are invertible.

$$X + AX = B$$

$\rightsquigarrow (I+A)X = B \rightsquigarrow X = (I+A)^{-1}B$

- Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. On the *left-hand side*, draw the null space of A. On the *right-hand side* draw the column space of A.





3. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation given by clockwise rotation by $\pi/4$ and let $U : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by the formula

$$U\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ x - y \end{pmatrix}$$

What is the standard matrix for T?

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

What is the standard matrix for U?

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

What is the range of U? $\mathbb{R}^2$

Which of the following two compositions makes sense? $T \circ U$ $U \circ T$

Write down the standard matrix for the composition you chose.

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

4. Consider the following matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} 3 & 6 & -1 \\ 4 & 8 & -1 \\ 4 & 8 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Let T be the matrix transformation $T(v) = Av$.

Find a basis for the null space of A.

$$\begin{aligned} x + 2y &= 0 \\ y &= y \\ z &= 0 \end{aligned} \quad \rightsquigarrow \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Find a basis for column space of A.

$$\left\{ \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix} \right\}$$

Are there two different vectors v and w with $T(v) = T(w)$ and $T(v) \neq 0$? YES NO

If you answered yes, find such a v and w. If you answered no, explain why not.

$$v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

5. Find the inverse of the following matrix.

$$\begin{pmatrix} 3 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 3 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 3 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -4 & 12 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right) \rightsquigarrow \begin{pmatrix} 0 & 1 & 0 \\ -4 & 12 & 1 \\ 1 & -3 & 0 \end{pmatrix}$$

Find a matrix A so the domain of $T(v) = Av$ is $\mathbb{R}^3$ and the range is the line $y = 2x$ in $\mathbb{R}^2$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$$

Find a 2×2 matrix A so that $A \neq I$ and so that $A^4 = I$. *Hint: Find a linear transformation* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *so that* $T \circ T \circ T \circ T$ *is the identity.*

$T =$ rotation $2\pi/4$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$