Name ________________________

Mathematics 1553

Quiz 2

Prof. Margalit

Section D1/Isabella D2/Kyle D3/Kalen D4/Sidhanth (circle one!)

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1. Which of the following matrices are in reduced row echelon form? Circle all that apply.

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \left(\begin{array}{c|c} 0 & 1 \\ 0 & 0 \end{array}\right) \qquad \left(\begin{array}{c|c} 0 & 0 \\ 0 & 1 \end{array}\right)$$

Answer:

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & | & 1 \\ 0 & | & 0 \end{pmatrix}$$

(All but the last one.)

Solution: Apply the definition of "reduced echelon form." The bar that separates the system coefficients from constants is irrelevant. Each of the first three satisfy the conditions, but not the last one (zero rows need to be below all nonzero rows).

2. Consider the following augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h & k \end{array}\right)$$

Find a choice for h and k so that the associated system of linear equations has infinitely many solutions.

Answer:

$$h = 0, k = 0$$

Solution: The matrix is in echelon form. For the system to have infinitely many solutions, it must be consistent and have at least one free variable. The system is inconsistent if there is a pivot in the augmented column; this occurs only if $h = 0$ and $k \neq 0$. Conversely, the system is consistent in two cases: (1) $h \neq 0$ and (2) $h = 0$ and $k = 0$. In the former, each column has a pivot, and the solution is unique. In the latter, the third column lacks a pivot, and there are infinitely many solutions.

Turn the page over!

Find the reduced row echelon form of the following (augmented) matrix

$$\left(\begin{array}{ccc|c} 0 & 2 & -10 & 0 \\ 2 & 3 & -9 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$$

Answer:

$$\begin{pmatrix} 1 & 0 & 3 & \mid & 0 \\ 0 & 1 & -5 & \mid & 0 \\ 0 & 0 & 0 & \mid & 0 \end{pmatrix}$$

Solution: For simplicity, below we will ignore the augmented column, since it will always be 0 under any sequence of row operations.

$$\begin{pmatrix} 0 & 2 & -10 \\ 2 & 3 & -9 \\ 1 & 1 & -2 \end{pmatrix} \xrightarrow{R_1 \longleftrightarrow R_3} \begin{pmatrix} 1 & 1 & -2 \\ 2 & 3 & -9 \\ 0 & 2 & -10 \end{pmatrix} \xrightarrow{-2R_1+R_2} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -5 \\ 0 & 2 & -10 \end{pmatrix} \xrightarrow{-2R_2+R_3} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$

This is in echelon form, but not in REF. We need one additional replacement to arrive at REF:

$$\begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{-R_2+R_1} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{pmatrix}.$$

Describe in parametric form the general solution to the system of equations:

$$\begin{aligned} 2x_2 - 10x_3 &= 0 \\ 2x_1 + 3x_2 \quad - 9x_3 &= 0 \\ x_1 \quad + x_2 \quad - 2x_3 &= 0 \end{aligned}$$

Answer:

$$\{(-3t, 5t, t) | t \in \mathbb{R}\}$$

Solution: Begin as usual by rewriting the solution as an augmented matrix:

$$\begin{pmatrix} 0 & 2 & -10 & \mid & 0 \\ 2 & 3 & -9 & \mid & 0 \\ 1 & 1 & -2 & \mid & 0 \end{pmatrix}$$

Note that this is exactly the matrix presented in the previous problem. Looking at the REF of the previous problem reveals that there is one free variable corresponding to the third column. Assign a parameter to it: $x_3 = t$. The REF matrix shows that $x_2 = 5t$ and $x_1 = -3t$. Therefore, the solution set to the given system is $\{(-3t, 5t, t) | t \in \mathbb{R}\}$.