Announcements Feb 4

- Midterm 2 on March 6
- WeBWorK 2.6 due Thursday
- My office hours Monday 3-4 and Wed 2-3
- TA office hours in Skiles 230 (you can go to any of these!)
  - Isabella Thu 2-3
  - Kyle Thu 1-3
  - Kalen Mon/Wed 1-1:50
  - Sidhanth Tue 10:45-11:45
- PLUS sessions Mon/Wed 6-7 LLC West with Miguel (different this week)
- Supplemental problems and practice exams on the master web site

Section 2.7

Bases

Bases

$V =$ subspace of $\mathbb{R}^n$

A basis for V is a set of vectors $\{v_1, v_2, \ldots, v_k\}$ such that

1. $V = \text{Span}\{v_1, \ldots, v_k\}$
2. $v_1, \ldots, v_k$ are linearly independent

Equivalently, a basis is a *minimal spanning set*, that is, a spanning set where if you remove any one of the vectors you no longer have a spanning set.

$\dim(V) =$ dimension of $V = k =$the number of vectors in the basis

(What is the problem with this definition of dimension?)

Q. What is one basis for $\mathbb{R}^2$? $\mathbb{R}^n$? How many bases are there?

Bases for $\mathbb{R}^n$

What are all bases for $\mathbb{R}^n$?

Take a set of vectors $\{v_1, \ldots, v_k\}$. Make them the columns of a matrix.

For the vectors to be linearly independent we need a pivot in every column.

For the vectors to span $\mathbb{R}^n$ we need a pivot in every row.

Conclusion: $k = n$ and the matrix has n pivots.

Who cares about bases

A basis $\{v_1, \ldots, v_k\}$ for a subspace V of $\mathbb{R}^n$ is useful because:

Every vector v in V can be written in exactly one way:

$$v = c_1 v_1 + \cdots + c_k v_k$$

So a basis gives coordinates for V, like latitude and longitude. See Section 2.8.

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

Find bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

Find bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Bases for $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$

In general:

- our usual parametric solution for $Ax = 0$ gives a basis for $\mathrm{Nul}(A)$
- the pivot columns of A form a basis for $\mathrm{Col}(A)$

Warning! Not the pivot columns of the reduced matrix.

What should you do if you are asked to find a basis for Span$\{v_1, \ldots, v_k\}$?

Bases for planes

Find a basis for the plane $2x + 3y + z = 0$ in $\mathbb{R}^3$.

Basis theorem

Basis Theorem

If V is a k-dimensional subspace of $\mathbb{R}^n$, then

- any k linearly independent vectors of V form a basis for V
- any k vectors that span V form a basis for V

In other words if a set has two of these three properties, it is a basis:

spans V, linearly independent, k vectors

We are skipping Section 2.8 this semester. But remember: the whole point of a basis is that it gives coordinates (like latitude and longitude) for a subspace. Every point has a unique address.

Section 2.7 Summary

- A basis for a subspace V is a set of vectors $\{v_1, v_2, \dots, v_k\}$ such that
  1. $V = \text{Span}\{v_1, \dots, v_k\}$
  2. $v_1, \dots, v_k$ are linearly independent
- The number of vectors in a basis for a subspace is the dimension.
- Find a basis for $\text{Nul}(A)$ by solving $Ax = 0$ in vector parametric form
- Find a basis for $\text{Col}(A)$ by taking pivot columns of A (not reduced A)
- Basis Theorem. Suppose V is a k-dimensional subspace of $\mathbb{R}^n$. Then
  - Any k linearly independent vectors in V form a basis for V.
  - Any k vectors in V that span V form a basis.