Announcements April 8

- Midterm 3 on April 17
- WeBWorK 5.5 & 5.6 due Thu Apr 9.
- Quiz 8am Fri - 8am Sat on 5.2 & 5.4.
  Expect less-Googlable questions.
- Survey about on-line learning on Canvas...
- My office hours Monday 3-4, Wed 2-3, and by appointment
- TA office hours on Blue Jeans (you can go to any of these!)
  - Isabella Wed 11-12
  - Kyle Wed 3-5, Thu 1-3
  - Kalen Mon/Wed 1-2
  - Sidhanth Tue 10-12
- Supplemental problems & practice exams on master web site
- Counseling Center: http://counseling.gatech.edu Click

Section 5.6

Stochastic Matrices (and Google!)

Outline of Section 5.6

- Stochastic matrices and applications
- The steady state of a stochastic matrix
- Important web pages

Stochastic matrices

A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.

Examples:

$$\begin{pmatrix} 1/4 & 3/5 \\ 3/4 & 2/5 \end{pmatrix} \quad \begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & 1 & 1/2 \\ 1/2 & 0 & 1/4 \\ 0 & 0 & 1/4 \end{pmatrix}$$

Application: Rental Cars

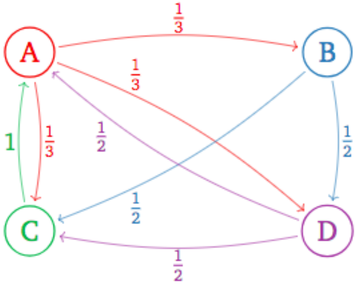
Say your car rental company has 3 locations. Make a matrix whose ij entry is the fraction of cars at location i that end up at location j. For example,

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Note the columns sum to 1. Why?

Application: Web pages

Make a matrix whose ij entry is the fraction of (randomly surfing) web surfers at page i that end up at page j. If page i has N links then the ij-entry is either 0 or $1/N$.



$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Properties of stochastic matrices

Let A be a stochastic matrix.

Fact. One of the eigenvalues of A is 1 and all other eigenvalues have absolute value at most 1.

Now suppose A is a positive stochastic matrix.

Fact. The 1-eigenspace of A is 1-dimensional; it has a positive eigenvector.

The unique such eigenvector with entries adding to 1 is called the steady state vector.

Fact. Under iteration, all nonzero vectors approach a multiple of the steady state vector. The multiple is the sum of the entries of the original vector.

Demo

The last fact tells us how to distribute rental cars, and also tells us the importance of web pages!

Application: Rental Cars

The rental car matrix is:

$$\begin{pmatrix} .3 & .4 & .5 \\ .3 & .4 & .3 \\ .4 & .2 & .2 \end{pmatrix}$$

Its steady state vector is:

$$\frac{1}{18}\begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix} \approx \begin{pmatrix} .39 \\ .33 \\ .28 \end{pmatrix}$$

Application: Web pages

The web page matrix is:

$$\begin{pmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{pmatrix}$$

Its steady state vector is approximately

$$\begin{pmatrix} .39 \\ .13 \\ .29 \\ .19 \end{pmatrix}$$

and so the first web page is the most important.

Review of Section 5.6

Can you make a stochastic matrix where the 1-eigenspace has dimension greater than 1?

Make your own internet and see if you can guess which web page is the most important. Check your answer using the method described in this section.

Summary of Section 5.6

- A stochastic matrix is a non-negative square matrix where all of the columns add up to 1.
- Every stochastic matrix has 1 as an eigenvalue, and all other eigenvalues have absolute value at most 1.
- A positive stochastic matrix has 1-dimensional eigenspace and has a positive eigenvector.
- For a positive stochastic matrix, a positive 1-eigenvector with entries adding to 1 is called a steady state vector.
- For a positive stochastic matrix, all nonzero vectors approach the steady state vector under iteration.
- Steady state vectors tell us the importance of web pages (for example).