Announcements Jan 8

- Mathematical autobiography due on Friday
- WeBWorK Warmup due Friday (not for a grade)
- Download the Piazza app
- My office hours today 2-3 and Monday 3-4 in Skiles 234
- Studio on Friday: same time, different room, with TA
- Remember the laptop rules

Section 1.1

Solving systems of equations

Outline of Section 1.1

- Learn what it means to solve a system of linear equations
- Describe the solutions as points in $\mathbb{R}^n$
- Learn what it means for a system of linear equations to be inconsistent

Solving equations

Solving equations

What does it mean to solve an equation?

$2x = 10$ A soln: x=5

$x + y = 1$ solns: x=1, y=0
x=0, y=1

$x + y + z = 0$ Solns: x=y=z=0 . . .

Find one solution to each. Can you find all of them?

A solution is a *list* of numbers. For example $(3, -4, 1)$.

Solving equations

What does it mean to solve a system of equations?

$$
\begin{aligned}
x + y &= 2 \\
y &= 1
\end{aligned}
$$

a soln: $x = 1$, $y = 1$

What about...

$$
\begin{aligned}
x + y + z &= 3 \\
x + y - z &= 1 \\
x - y + z &= 1
\end{aligned}
$$

yes   no — 3^{rd} eqn.

Is $(1, 1, 1)$ a solution? Is $(2, 0, 1)$ a solution? What are all the solutions?

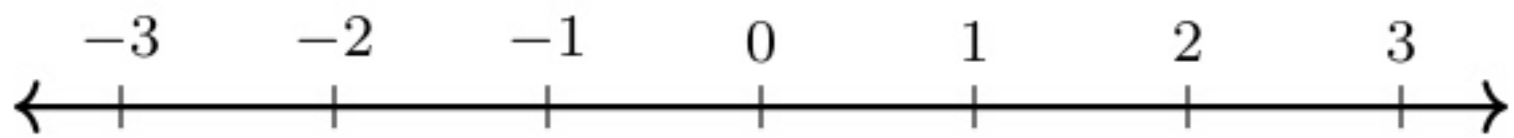
Soon, you will be able to see just by looking that there is exactly one solution.

$$\mathbb{R}^n$$

$\mathbb{R}^n$

$\mathbb{R}$ = denotes the set of all real numbers

Geometrically, this is the *number line*.



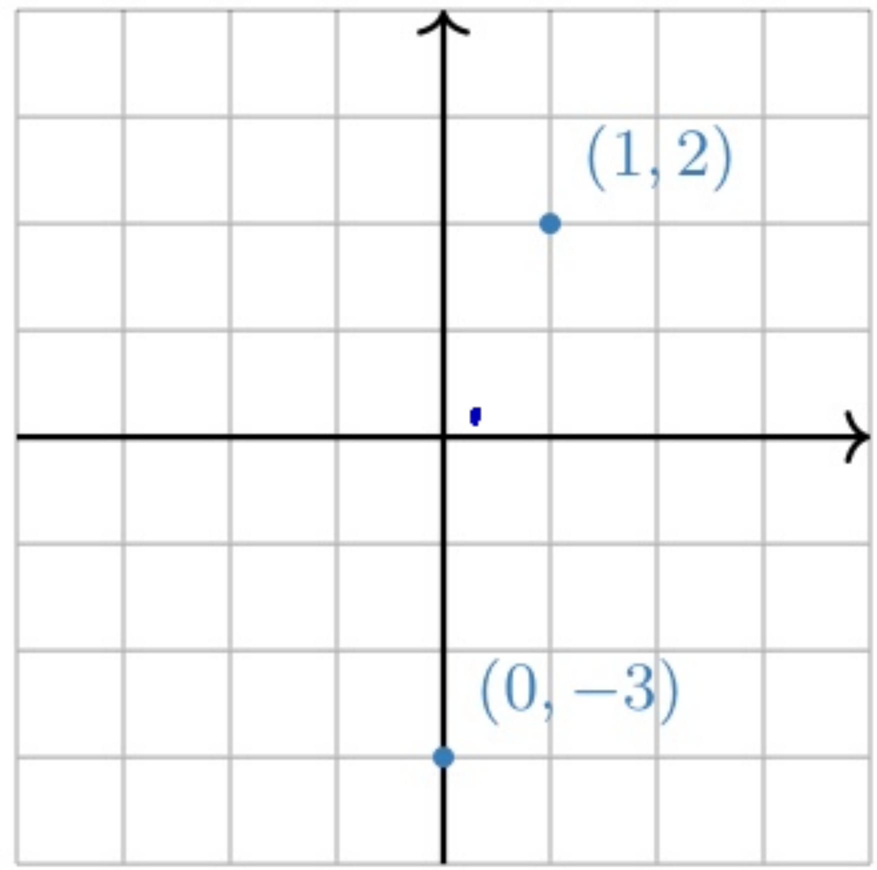
$\mathbb{R}^n$ = all ordered n-tuples (or lists) of real numbers $(x_1, x_2, x_3, \ldots, x_n)$

Solutions to systems of equations are exactly points in $\mathbb{R}^n$.

A point in $\mathbb{R}^2$:

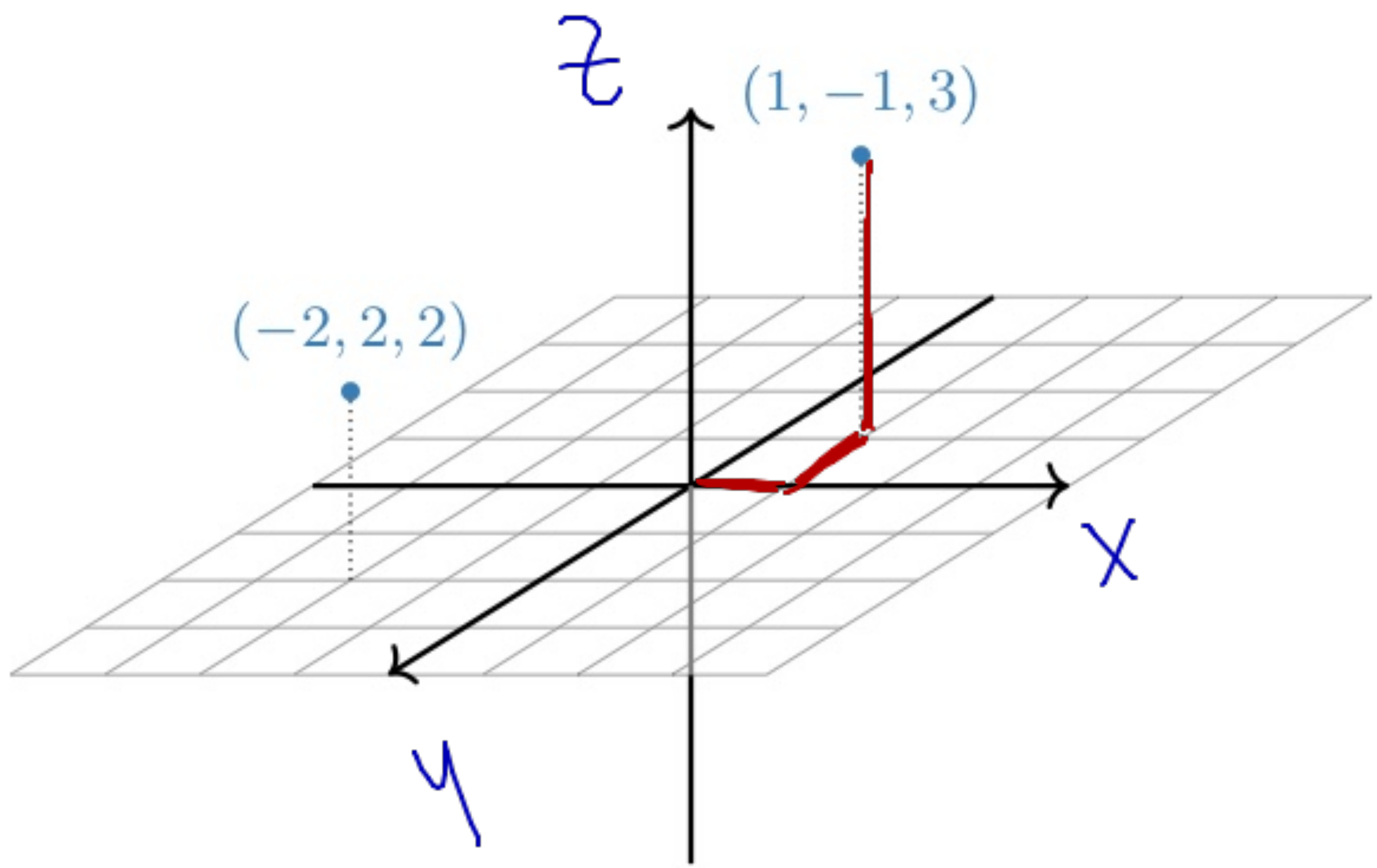
$(1, 1)$ or $(\pi, \sqrt{2}+e)$

When $n = 2$, we can visualize of $\mathbb{R}^2$ as the *plane*.

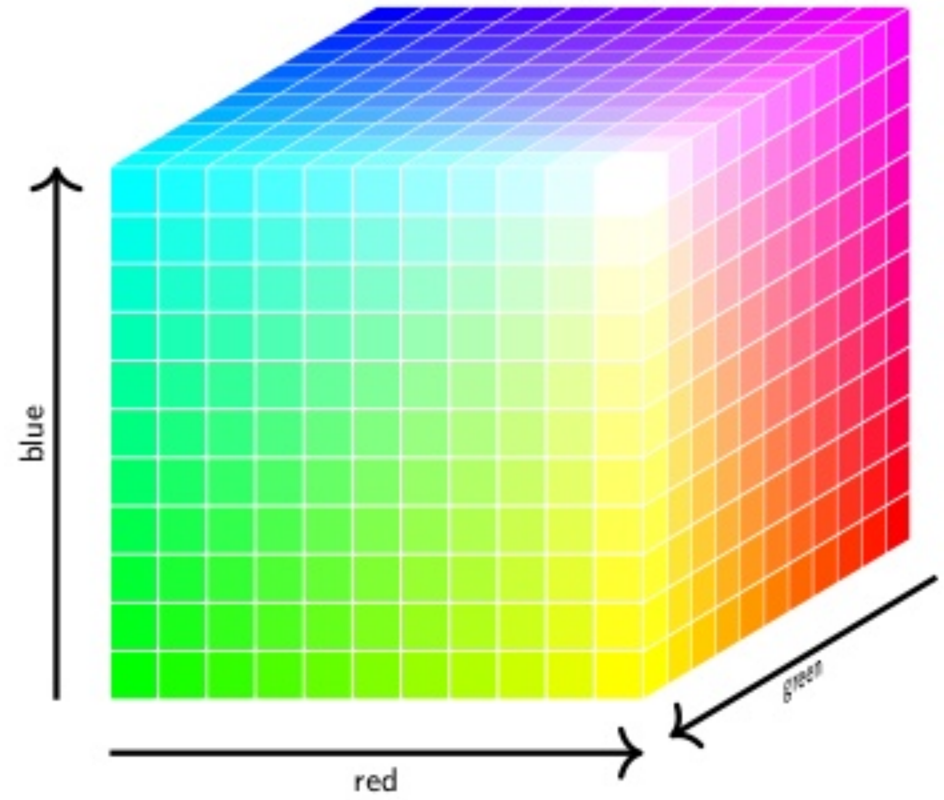


$\mathbb{R}^n$

When $n = 3$, we can visualize $\mathbb{R}^3$ as the *space* we (appear to) live in.



We can think of the space of all *colors* as (a subset of) $\mathbb{R}^3$:



So what is $\mathbb{R}^4$? or $\mathbb{R}^5$? or $\mathbb{R}^n$?

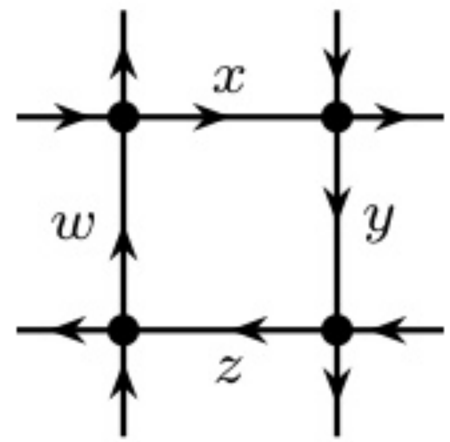
...go back to the *definition*: ordered n-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbb{R}^2$ and $\mathbb{R}^3$ sometimes extends to $\mathbb{R}^n$, but they're harder to visualize.

$\mathbb{R}^n$

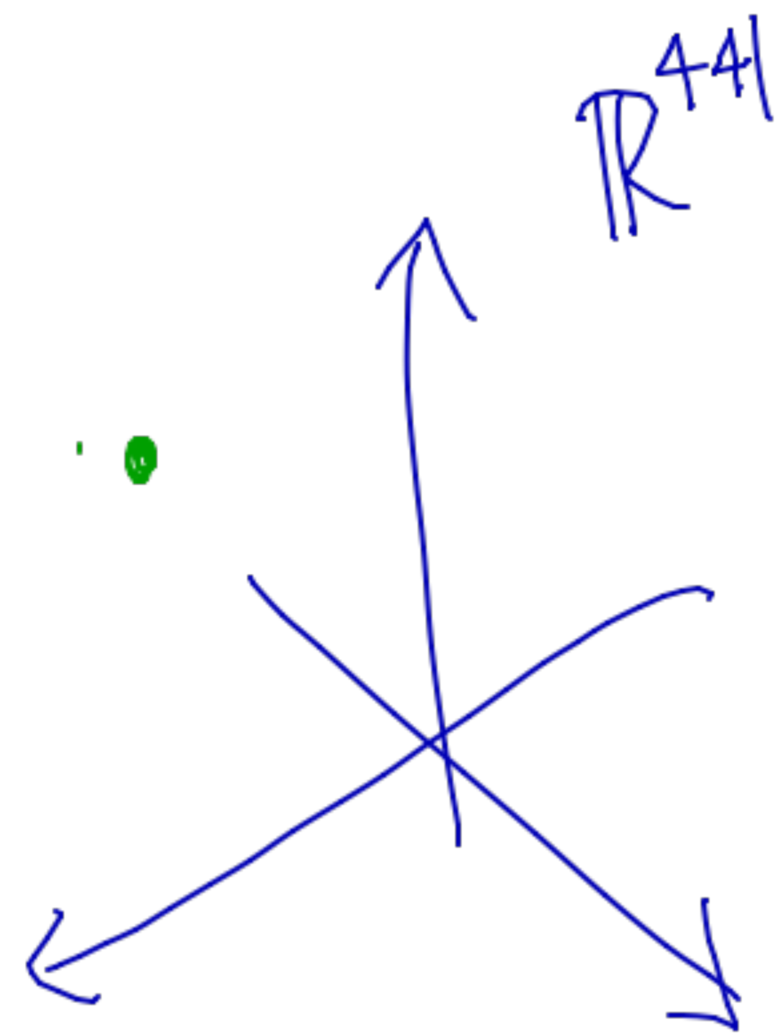
Last time we could have used $\mathbb{R}^4$ to label the amount of traffic (x, y, z, w) passing through four streets.



We'll make definitions and state theorems that apply to any $\mathbb{R}^n$, but we'll only draw pictures in $\mathbb{R}^2$ and $\mathbb{R}^3$.

$\mathbb{R}^n$ and QR codes

This is a 21×21 QR code. We can also think of this as an element of $\mathbb{R}^n$.



How? Which n?

What about a greyscale image?

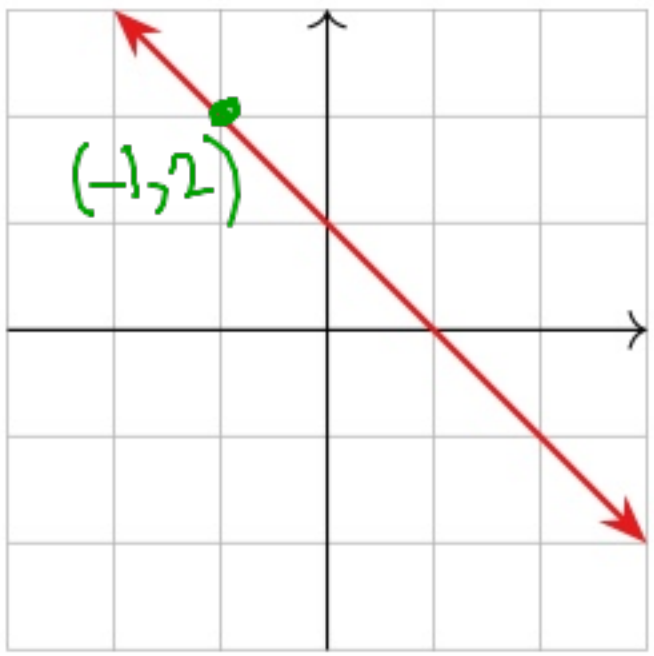
This is a powerful idea: instead of thinking of a QR code as 441 pieces of information, we think of it as one piece of information.

Visualizing solutions: a preview

One Linear Equation

What does the solution set of a linear equation look like?

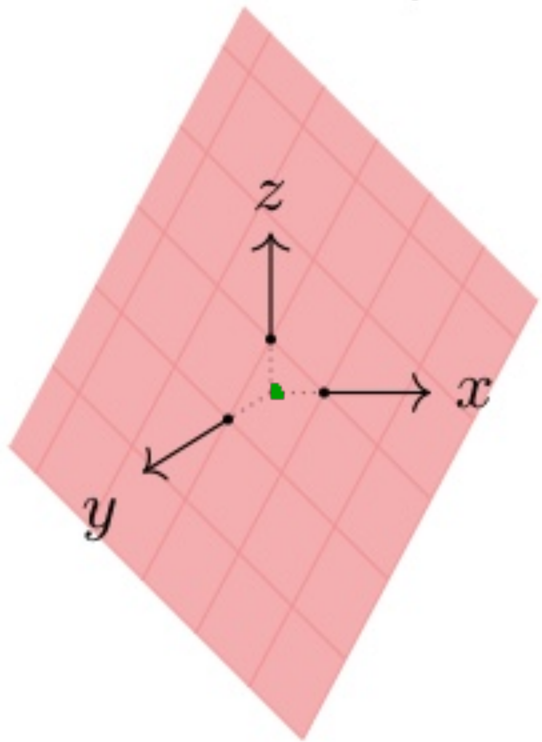
$x + y = 1 \rightsquigarrow$ a line in the plane: $y = 1 - x$



One Linear Equation

What does the solution set of a linear equation look like?

$x + y + z = 1 \rightsquigarrow$ a plane in space:



One Linear Equation

Continued

What does the solution set of a linear equation look like?

$x + y + z + w = 1 \rightsquigarrow$ a "3-plane" in "4-space"...

1 eqn in 100 vars:

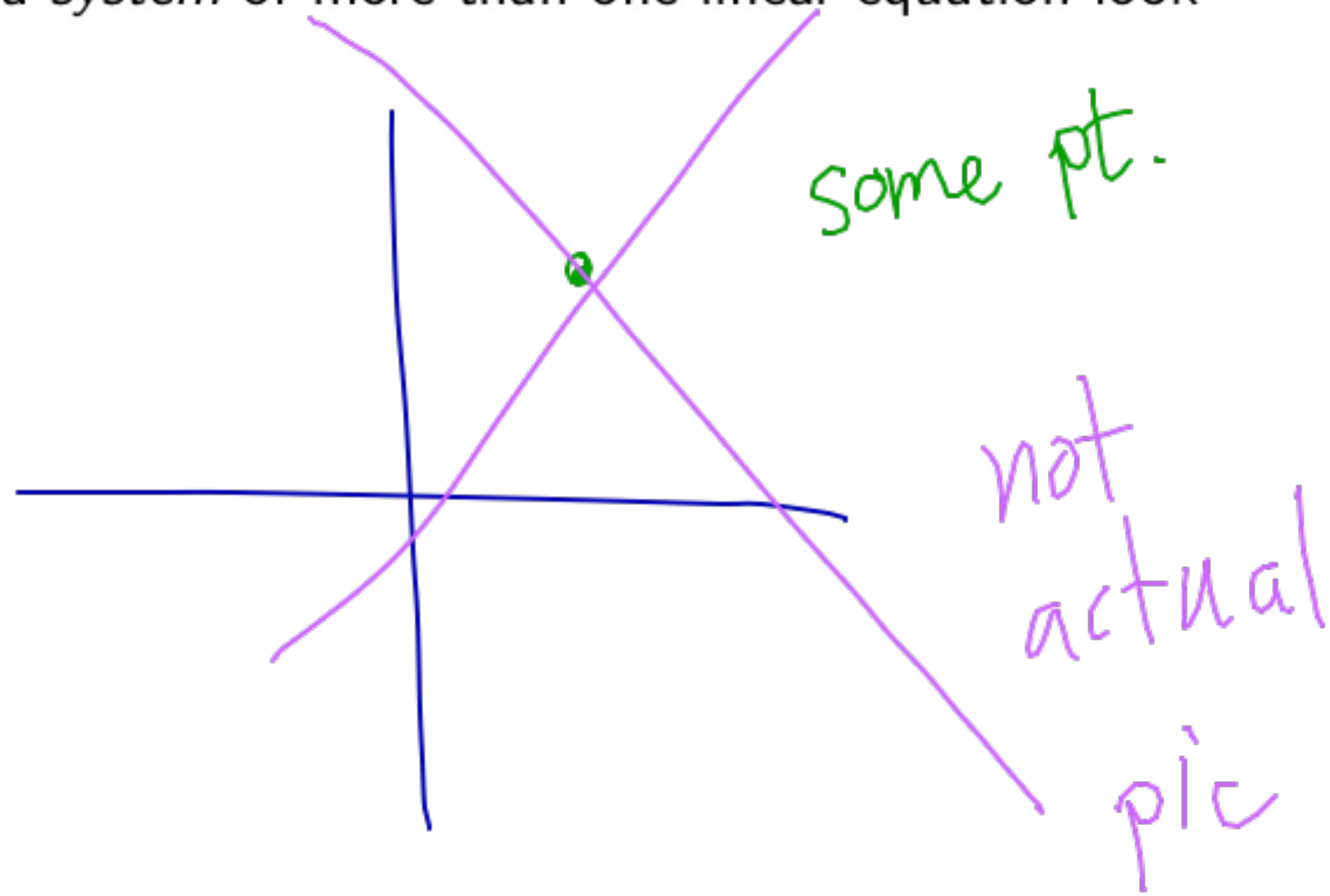
99 dim plane

in $\mathbb{R}^{100}$

Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

$$x - 3y = -3$$
$$2x + y = 8$$



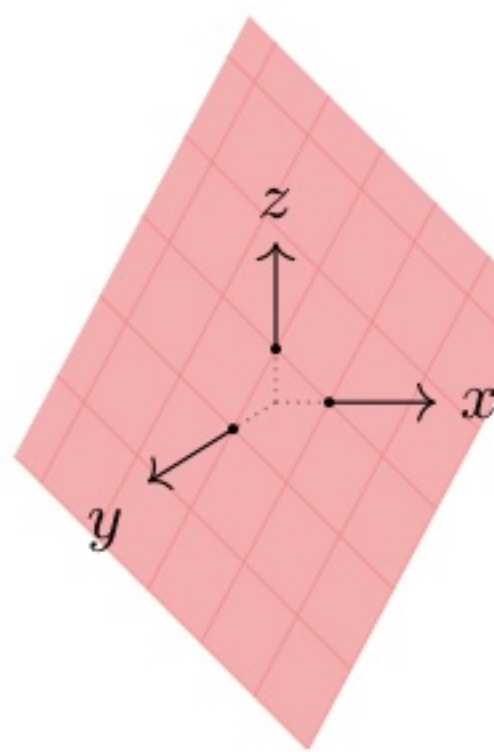
What are the other possibilities for two equations with two variables?

What if there are more variables? More equations?

Poll

Is the plane in $\mathbb{R}^3$ from the previous example equal to $\mathbb{R}^2$? What about the xy-plane in $\mathbb{R}^3$?

1. yes + yes
2. yes + no
3. no + yes
4. no + no



Consistent versus Inconsistent

We say that a system of linear equations is consistent if it has a solution and inconsistent otherwise.

$$\begin{aligned} x + y &= 1 \\ x + y &= 2 \end{aligned}$$

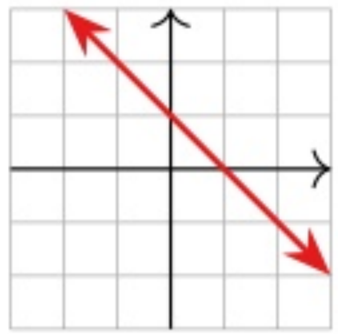
Why is this inconsistent?

~~1 = 2~~

What are other examples of inconsistent systems of linear equations?

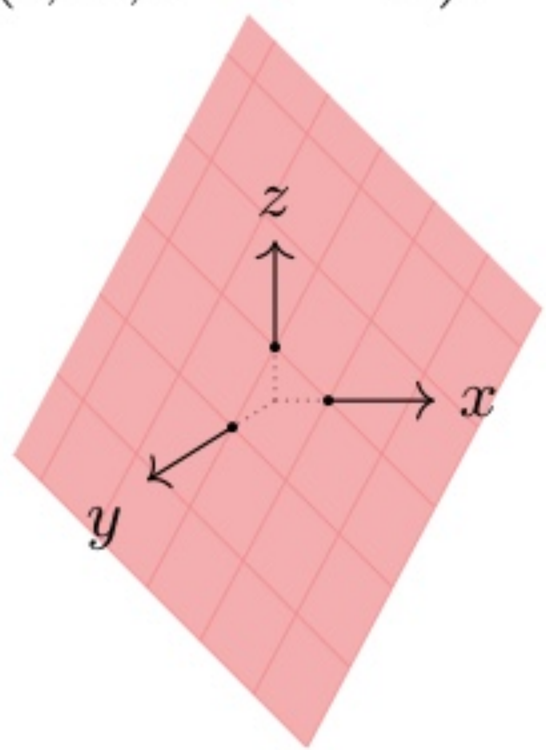
Parametric form

The equation $y = 1 - x$ is an implicit equation for the line in the picture.



It also has a parametric form: $(t, 1-t)$

Similarly the equation $x + y + z = 1$ is an implicit equation. One parametric form is: $(t, w, 1-t-w)$.



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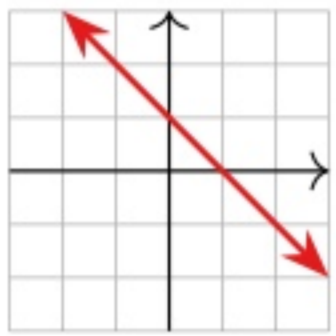

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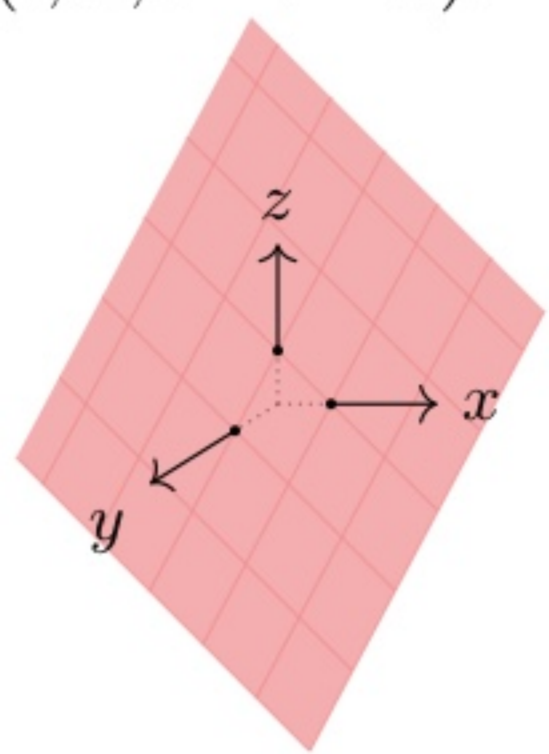

Imp: $z = 0$

Param: $(t, w, 0)$

What is an implicit equation and a parametric form for the xy-plane in $\mathbb{R}^3$?

Summary of Section 1.1

- A solution to a system of linear equations in n variables is a point in $\mathbb{R}^n$.
- The set of all solutions to a single equation in n variables is an $(n-1)$-dimensional plane in $\mathbb{R}^n$
- The set of solutions to a system of m linear equations in n variables is the intersection of m of these $(n-1)$-dimensional planes in $\mathbb{R}^n$.
- A system of equations with no solutions is said to be inconsistent.
- Line and planes have implicit equations and parametric forms.