Announcements Mar 25

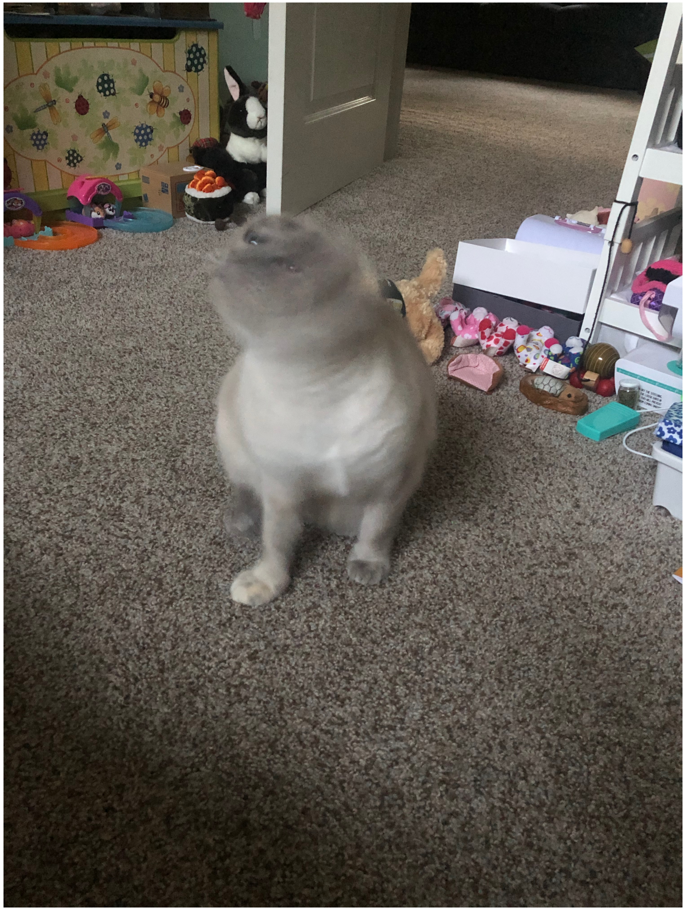
- Class participation (Piazza polls) is optional for the rest of the semester.
- We will use Blue Jeans Meetings for the rest of the semester.
- The new schedule will be released March 30.
- Midterm 3 on April 17
- WeBWorK 5.1 due Thu April 2.
- Practice quiz is open until Wed at 5. You have 25 minutes once you start. It is not for a grade.
- Official quiz next Friday on Canvas. It will be open all day Friday, but there will be a time limit.
- Lights out questions on Piazza!
- My office hours Monday 3-4 and Wed 2-3 on Blue Jeans starting next week
- TA office hours on Blue Jeans (you can go to any of these!)
  - Isabella Mon 11-12, Wed 11-12
  - Kyle Wed 3-6, Thu 1-4
  - Kalen Mon/Wed 1-2
  - Sidhanth Tue 10-12
- Supplemental problems and practice exams on the master web site

I made a video

The cat

The cat

The cat

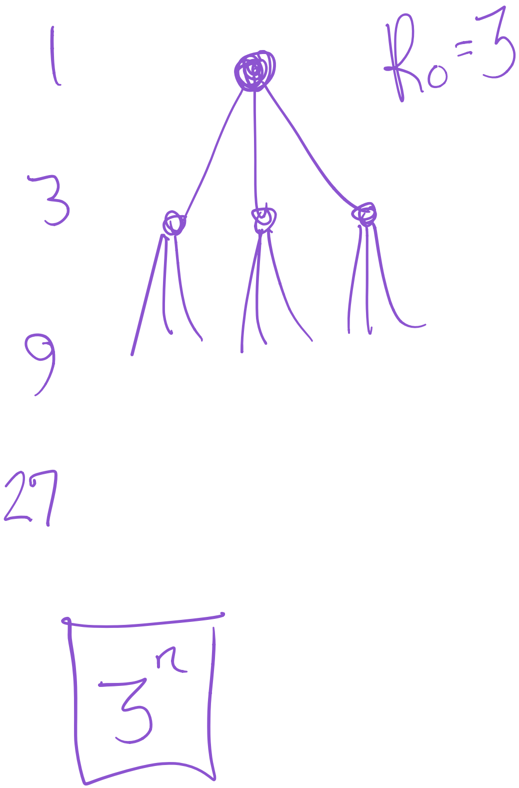


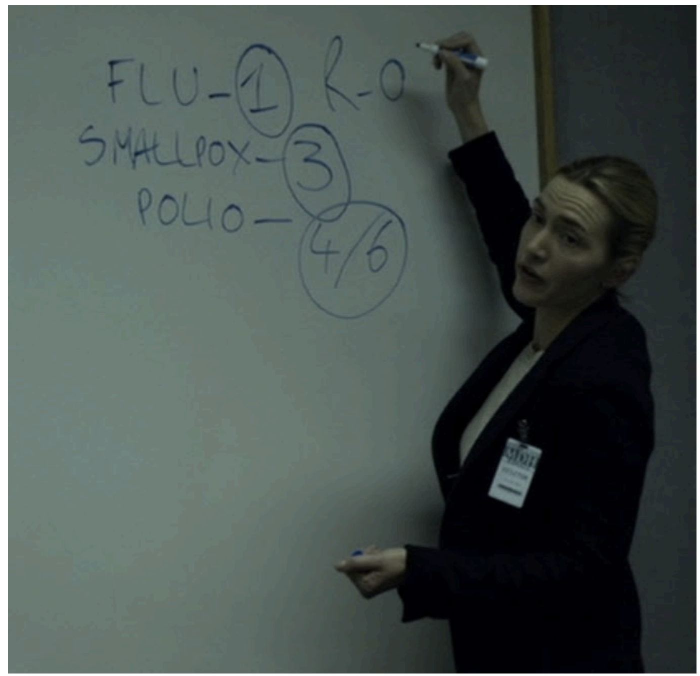
The cat

MARION
COTILLARD
MATT
DAMON
LAURENCE
FISHBURNE
JUDE
LAW
GWYNETH
PALTROW
KATE
WINSLET
NOTHING SPREADS LIKE FEAR
CONTAGION

R_0

For a given virus, R_0 is the average number of people that each infected person infects. If R_0 is large, that is bad. Patient zero infects R_0 people, who then infect R_0^2 people, who then infect R_0^3 people. That is exponential growth. (If R_0 is less than 1, then that's good.)





Eigenvectors and Eigenvalues

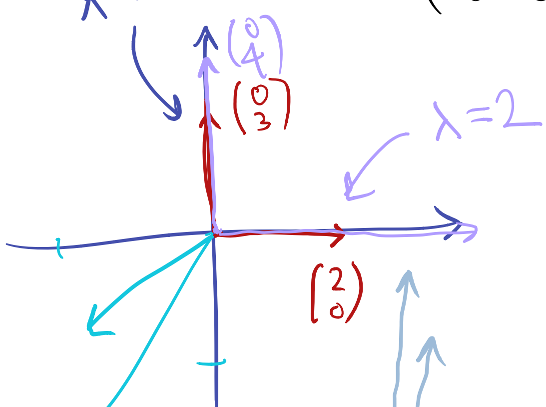
Suppose A is an $n \times n$ matrix and there is a $v \neq 0$ in $\mathbb{R}^n$ and λ in $\mathbb{R}$ so that

$$Av = \lambda v$$

then v is called an eigenvector for A, and λ is the corresponding eigenvalue.

Can you find any eigenvectors/eigenvalues for the following matrix?

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$



What happens when you apply larger and larger powers of A to a vector?

Eigenvectors and Eigenvalues

not x-axis. ↑ another eigenspace.

So for the matrix

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

we see that if we take (almost) any vector v and apply powers of A...

$$v, Av, A^2v, A^3v, \ldots$$

then eventually the vectors are pointing (almost) vertically, and the lengths multiply by (almost) 3 every time.

So the lengths of the vectors $A^k v$ grow like 3^k (exponential growth).

This is happening because 3 is the largest eigen~~vector~~value and the ~~$\binom{0}{1}$ is its eigenvector.~~ ~~one of~~ y-axis is its eigenspace.

A Question from Biology

In a population of rabbits...

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population one year - think of it as a vector - what is the population the next year?

$$A\begin{pmatrix} f \\ s \\ t \end{pmatrix}$$

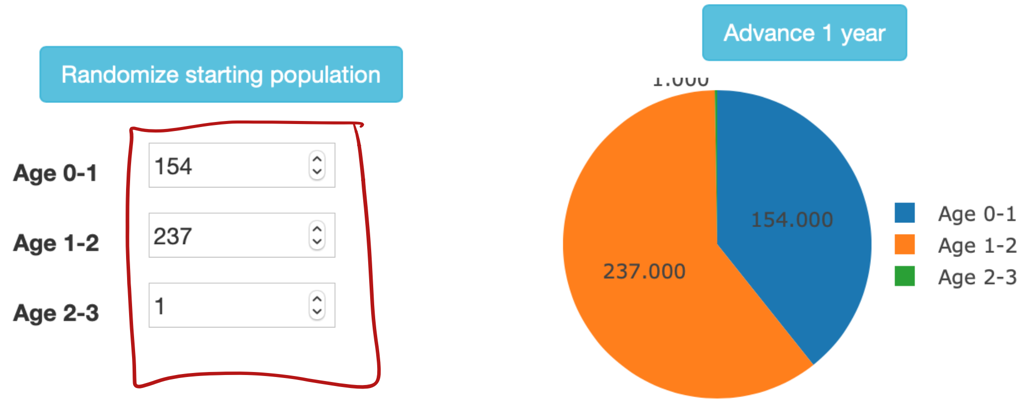
Answer. apply this matrix:

$$A = \begin{pmatrix} 0 & 6 & 8 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

Now choose some starting population vector u and choose some number of years N. What is the new population after N years?

Demo

Rabbits

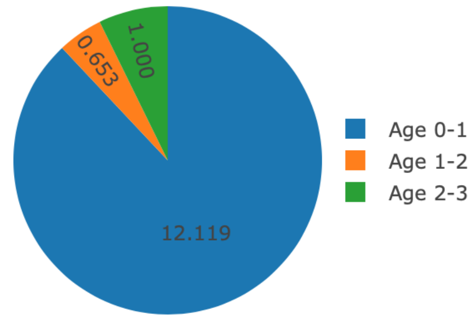

Randomize starting population
Age 0-1
154
Age 1-2
237
Age 2-3
1
Advance 1 year
1.000
154.000
237.000
Age 0-1
Age 1-2
Age 2-3

Rabbits

Randomize starting population

Age 0-1	1430
Age 1-2	77
Age 2-3	118

Advance 1 year

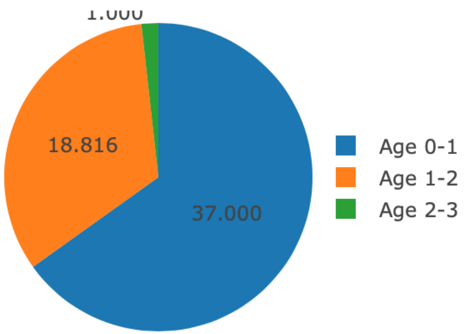


Rabbits

Randomize starting population

Age 0-1	1406
Age 1-2	715
Age 2-3	38

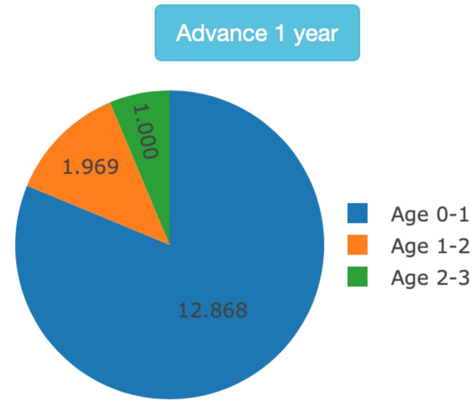
Advance 1 year



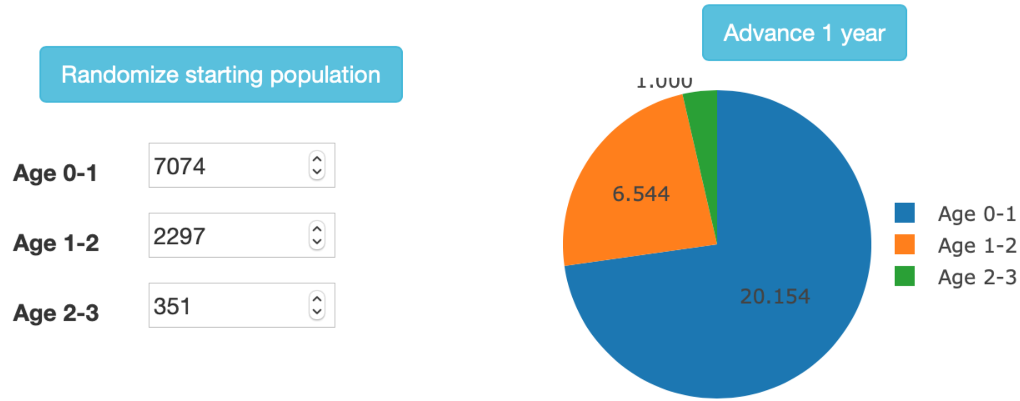
Rabbits

Randomize starting population

Age 0-1	4594
Age 1-2	703
Age 2-3	357



Rabbits

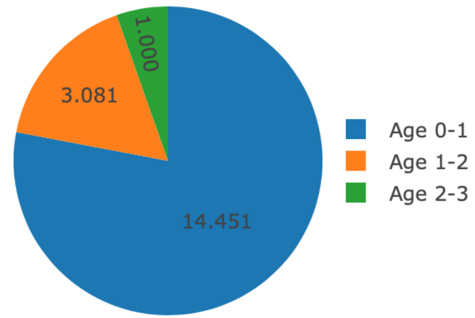

Randomize starting population
Age 0-1
7074
Age 1-2
2297
Age 2-3
351
Advance 1 year
1.000
6.544
20.154
Age 0-1
Age 1-2
Age 2-3

Rabbits

Randomize starting population

Age 0-1	16590
Age 1-2	3537
Age 2-3	1148

Advance 1 year

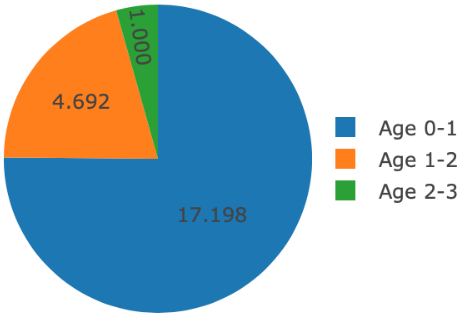


Rabbits

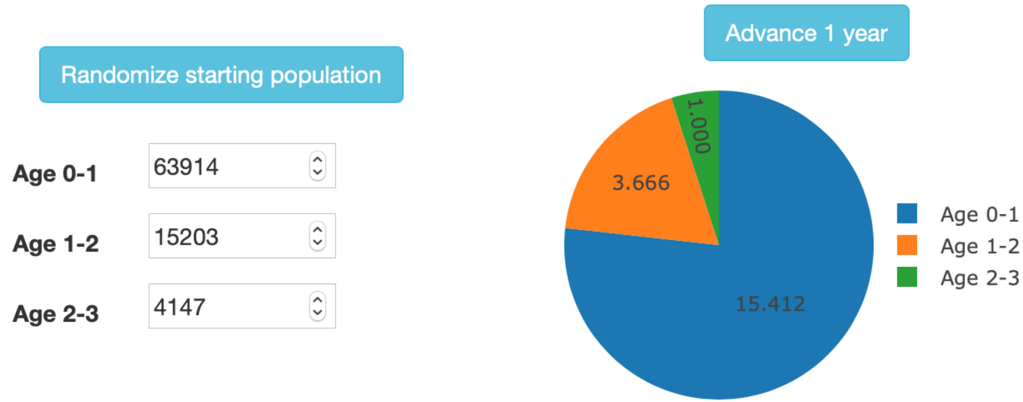
Randomize starting population

Age 0-1	30406
Age 1-2	8295
Age 2-3	1768

Advance 1 year

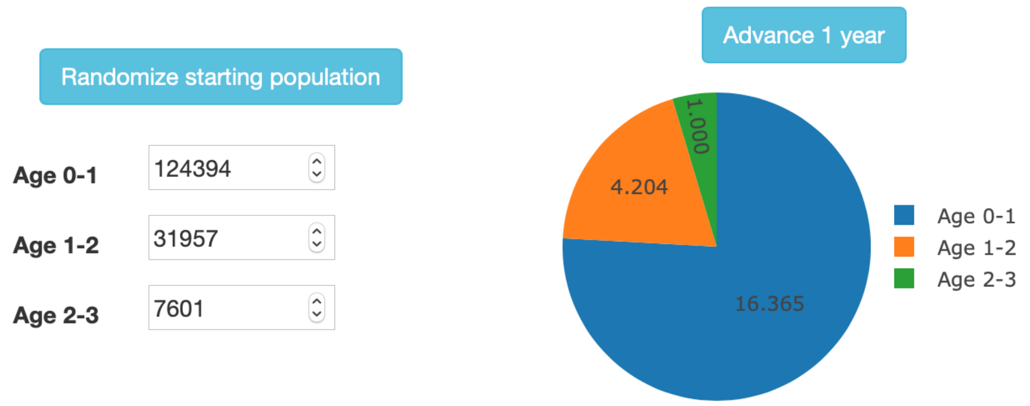


Rabbits



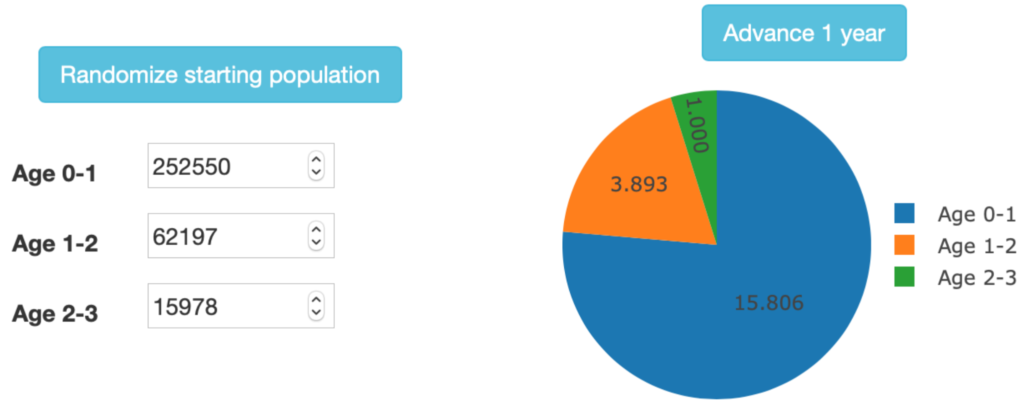
Randomize starting population
Age 0-1
63914
Age 1-2
15203
Age 2-3
4147
Advance 1 year
1.000
3.666
15.412
Age 0-1
Age 1-2
Age 2-3

Rabbits

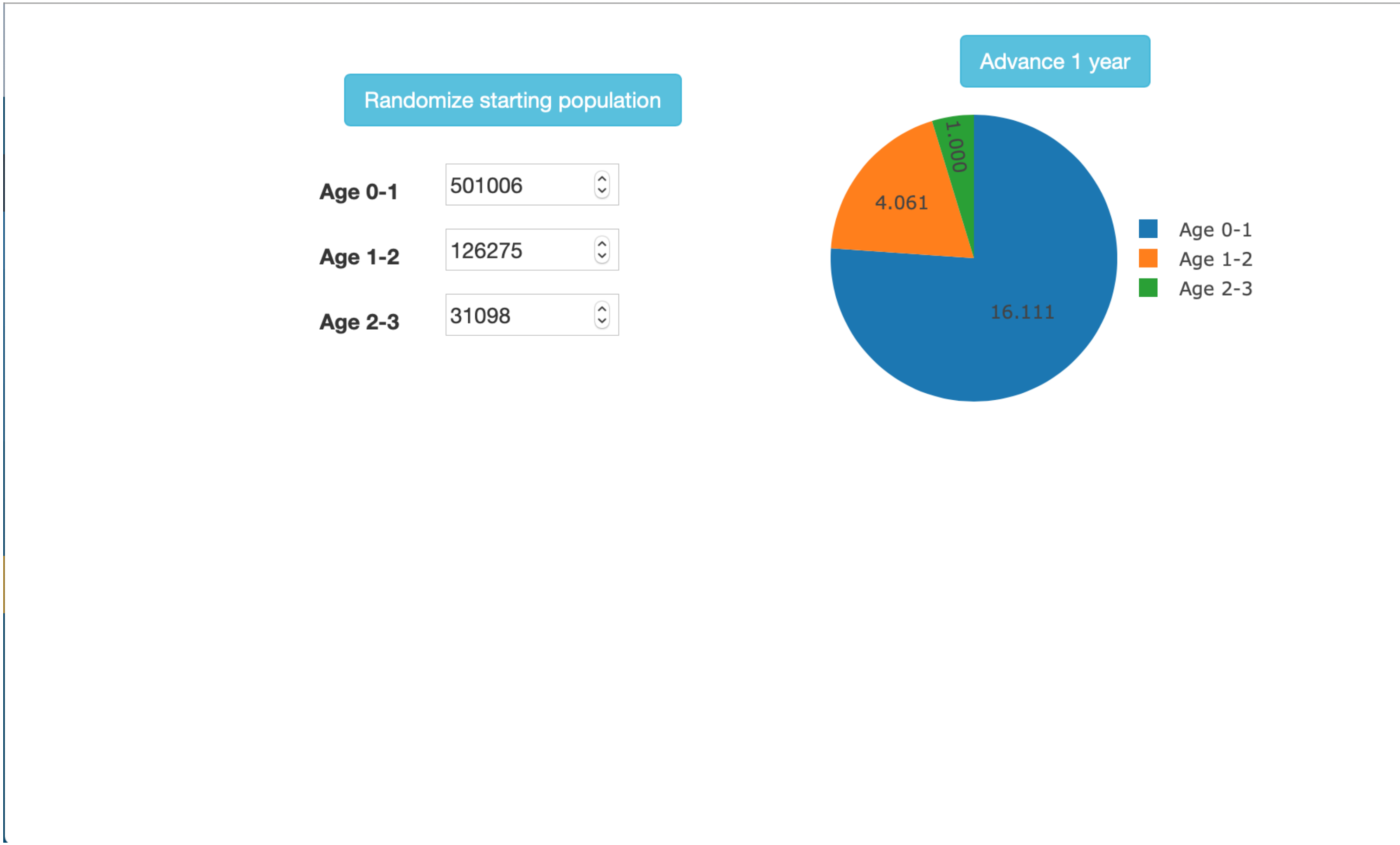


Randomize starting population
Age 0-1
124394
Age 1-2
31957
Age 2-3
7601
Advance 1 year
1.000
4.204
16.365
Age 0-1
Age 1-2
Age 2-3

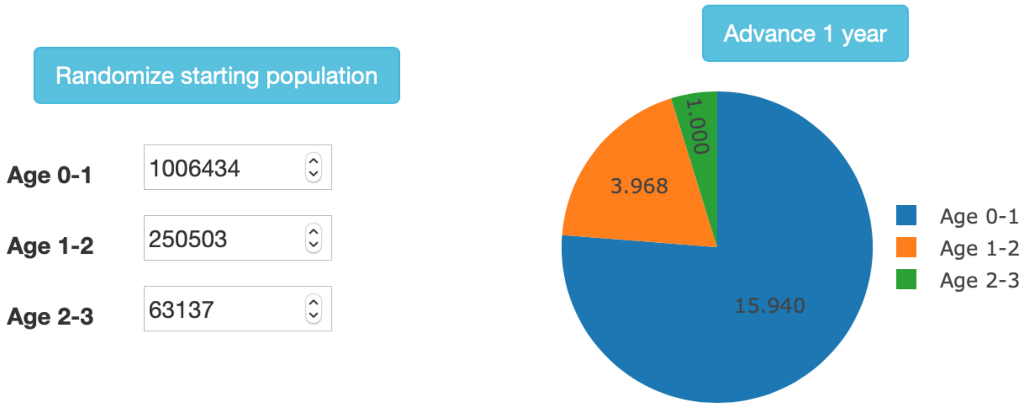
Rabbits


Randomize starting population
Age 0-1
252550
Age 1-2
62197
Age 2-3
15978
Advance 1 year
1.000
3.893
15.806
Age 0-1
Age 1-2
Age 2-3

Rabbits


Randomize starting population
Age 0-1
501006
Age 1-2
126275
Age 2-3
31098
Advance 1 year
1.000
4.061
16.111
Age 0-1
Age 1-2
Age 2-3

Rabbits

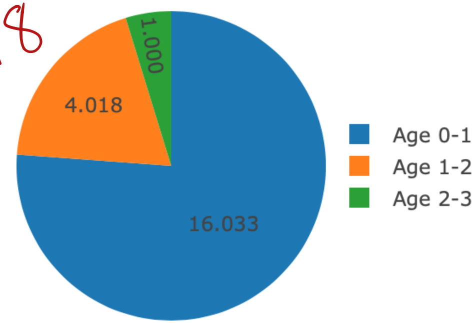

Randomize starting population
Age 0-1
1006434
Age 1-2
250503
Age 2-3
63137
Advance 1 year
1.000
3.968
15.940
Age 0-1
Age 1-2
Age 2-3

Rabbits

Randomize starting population

Age 0-1	2008114
Age 1-2	503217
Age 2-3	125251

Advance 1 year



1

4.018

16.033

doubled from prev. year

(largest) eigenvalue 2

eigenvector $\begin{pmatrix} 16 \\ 4 \\ 1 \end{pmatrix}$

If I tell you $\lambda = 2$
Find eigenvectors by solving
$(A - 2I)v = 0$

Eigenvectors and Eigenvalues

not the eigenvectors for other eigenvalue!

So for the matrix

$$A = \begin{pmatrix} 0 & 6 & 8 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

we see that if we take (almost) any vector v and apply powers of A...

$$v, Av, A^2v, A^3v, \ldots$$

then eventually the vectors are pointing (almost) in the direction $\begin{pmatrix} 16 \\ 4 \\ 1 \end{pmatrix}$, and the lengths - or, total population - multiplies by (almost) 2 every time.

So the lengths of the vectors - or, total polulation - $A^k v$ grow like 2^k (exponential growth). That means it doubles every year.

Also, the ratio of first:second:third year rabbits approaches 16:4:1.

This is happening because 2 is the largest eigen~~vector~~value and $\begin{pmatrix} 16 \\ 4 \\ 1 \end{pmatrix}$ is its eigenvector. (the span of (16,4,1) is the eigenspace)

R_0

For a given virus, R_0 is the average number of people that each infected person infects. If R_0 is large, that is bad. Patient zero infects R_0 people, who then infect R_0^2 people, who then infect R_0^3 people. That is exponential growth.

Whenever we see an exponential growth rate, we should think: eigenvalue.

It turns out that R_0 is an eigenvalue. The rough idea is very similar to our rabbit example: split the population into compartments, figure out how often each compartment infects each other compartment. That's a matrix. The largest eigenvalue is R_0.

R_0 is an eigenvalue

It turns out that R_0 is an eigenvalue. The rough idea is very similar to our rabbit example: split the population into compartments, figure out how often each compartment infects each other compartment.

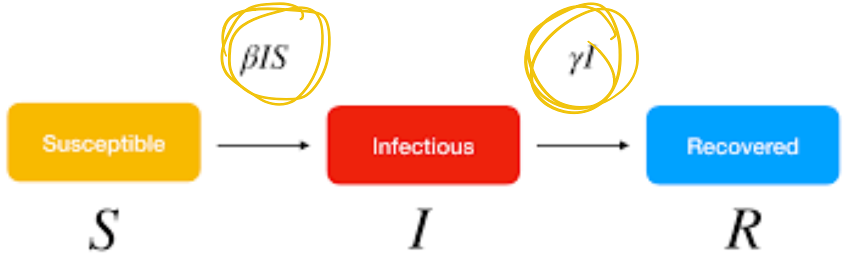
For malaria, the compartments might be mosquitoes and humans. 2x2 matrix

For a sexually transmitted disease in a heterosexual population, the compartments might be males and females.

R_0 is an eigenvalue

It turns out that R_0 is an eigenvalue. The rough idea is very similar to our rabbit example: split the population into compartments, figure out how often each compartment infects each other compartment.

The SIR model has compartments for Susceptible, Infected, and Recovered.



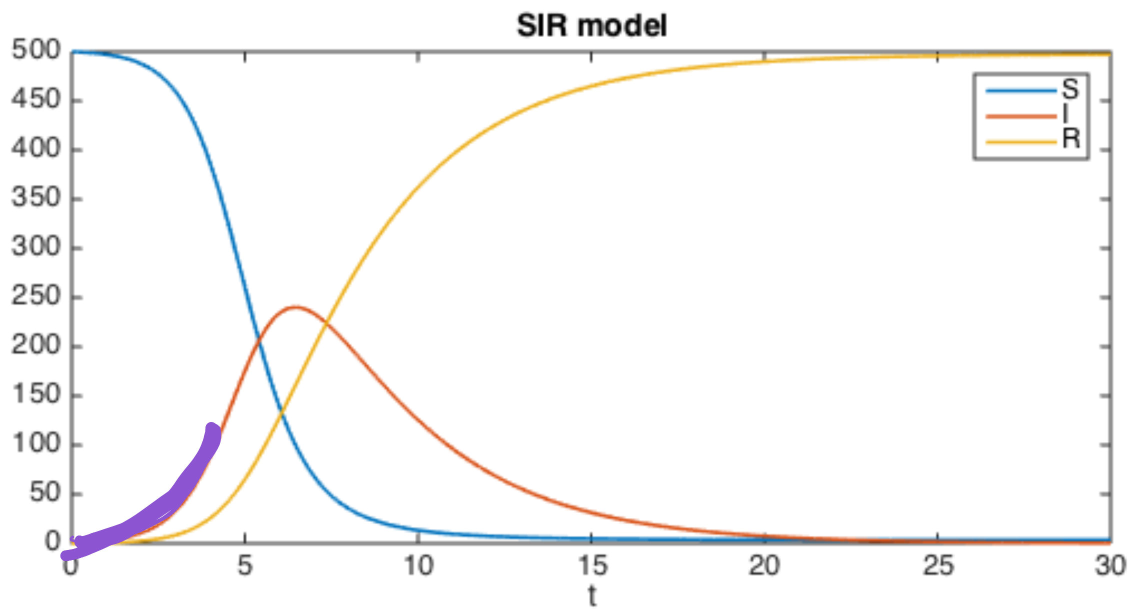
The arrows are governed by differential equations (Math 2552). Why do the labels on the arrows make sense? (The greek letters are constants).

There is a nice discussion of this by James Holland Jones (Stanford).

▸ Paper

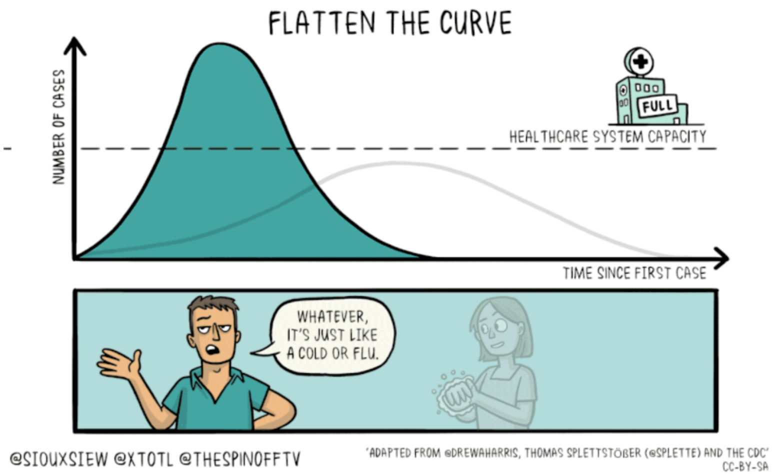
Bell curves

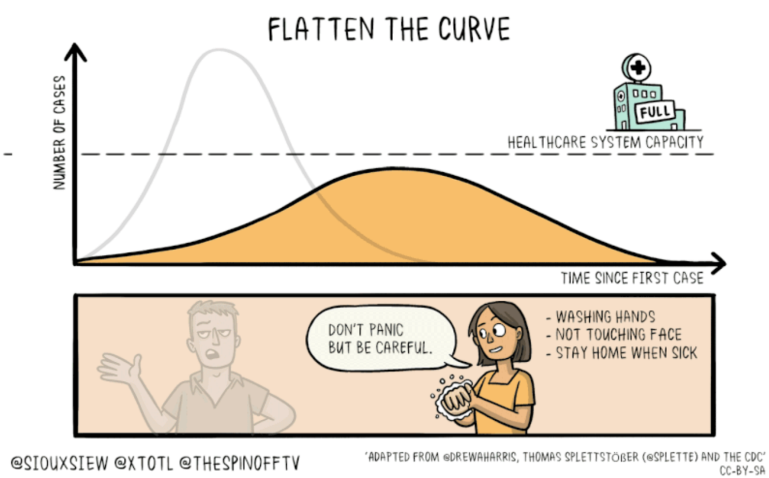
The growth rate of infection does not stay exponential forever, because the recovered population has immunity. That's where you get these bell curves.



Public Service Announcement

Social distancing decreases R_0





Keep learning. You guys will be the solution.