Announcements Mar 4

- Midterm 2 on Friday
- WeBWorK 3.4, 3.5, and 3.6 due Thursday
- TA office hours in Skiles 230 (you can go to any of these!)
  - Isabella Thu 2-3
  - Kyle Thu 1-3
  - Kalen Mon/Wed 1-1:50
  - Sidhanth Tue 10:45-11:45
- Review sessions
  - Kalen 7 pm Thu online
  - Sidhanth 7 pm tonite
- PLUS session with Miguel tonight 5-7
- Supplemental problems and practice exams on the master web site

Review for Midterm 2

Section 2.6 Summary

- A subspace of $\mathbb{R}^n$ is a subset V with:
  1. The zero vector is in V.
  2. If u and v are in V, then $u + v$ is also in V.
  3. If u is in V and c is in $\mathbb{R}$, then $cu \in V$.
- Two important subspaces: $\mathrm{Nul}(A)$ and $\mathrm{Col}(A)$
- Find a spanning set for $\mathrm{Nul}(A)$ by solving $Ax = 0$ in vector parametric form
- Find a spanning set for $\mathrm{Col}(A)$ by taking pivot columns of A (not reduced A)
- Four things are the same: subspaces, spans, planes through 0, null spaces

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Let V be the subset of $\mathbb{R}^3$ consisting of the x-axis, the y-axis, and the z-axis. Which properties of a subspace does V fail?

Find a spanning set for the plane in $\mathbb{R}^3$ defined by $x + y - 2z = 0$.

Section 2.7 Summary

- A basis for a subspace V is a set of vectors $\{v_1, v_2, \ldots, v_k\}$ such that
  1. $V = \text{Span}\{v_1, \ldots, v_k\}$
  2. $v_1, \ldots, v_k$ are linearly independent
- The number of vectors in a basis for a subspace is the dimension.
- Find a basis for $\text{Nul}(A)$ by solving $Ax = 0$ in vector parametric form
- Find a basis for $\text{Col}(A)$ by taking pivot columns of A (not reduced A)
- Basis Theorem. Suppose V is a k-dimensional subspace of $\mathbb{R}^n$. Then
  - Any k linearly independent vectors in V form a basis for V.
  - Any k vectors in V that span V form a basis.

Find a basis $\{u, v, w\}$ for $\mathbb{R}^3$ where no vector has a zero entry.

Section 2.9 Summary

- Rank-Nullity Theorem. $\text{rank}(A) + \dim \text{Nul}(A) = \#\text{cols}(A)$

Let A be an 4×6 nonzero matrix and suppose the columns of A are all the same. What is $\dim \text{Nul}(A)$?

Section 3.1 Summary

- If A is an $m \times n$ matrix, then the associated matrix transformation T is given by $T(v) = Av$. This is a function with domain $\mathbb{R}^n$ and codomain $\mathbb{R}^m$ and range $\mathrm{Col}(A)$.
- If A is $n \times n$ then T does something to $\mathbb{R}^n$; basic examples: reflection, projection, scaling, shear, rotation

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Find a matrix A so that the range of the matrix transformation $T(v) = Av$ is the line $y = 2x$ in $\mathbb{R}^2$.

Summary of Section 3.2

- $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each b in $\mathbb{R}^m$ is the output for at most one v in $\mathbb{R}^n$.
- **Theorem.** Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation with matrix A. Then the following are all equivalent:
  - T is one-to-one
  - the columns of A are
  - $Ax = 0$ has
  - A has a pivot
  - the range has dimension n
- $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto if the range of T equals the codomain $\mathbb{R}^m$, that is, each b in $\mathbb{R}^m$ is the output for at least one input v in $\mathbb{R}^m$.
- **Theorem.** Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation with matrix A. Then the following are all equivalent:
  - T is onto
  - the columns of A
  - A has a pivot
  - $Ax = b$ is consistent
  - the range of T has dimension m

Let A be an 5×5 matrix. Suppose that $\dim \mathrm{Nul}(A) = 0$. Must it be true that $Ax = e_1$ is consistent?

Summary of Section 3.3

- A function $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear if
  - $T(u+v) = T(u) + T(v)$ for all u, v in $\mathbb{R}^n$.
  - $T(cv) = cT(v)$ for all $v \in \mathbb{R}^n$ and c in $\mathbb{R}$.
- **Theorem.** Every linear transformation is a matrix transformation (and vice versa).
- The standard matrix for a linear transformation has its ith column equal to $T(e_i)$.

Find the standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects over the line $y = -x$ and then rotates counterclockwise by $\pi/2$.

Summary of Section 3.4

- Composition: $(T \circ U)(v) = T(U(v))$ (do U then T)
- Matrix multiplication: $(AB)_{ij} = r_i \cdot b_j$
- Matrix multiplication: the ith column of AB is $A(b_i)$
- The standard matrix for a composition of linear transformations is the product of the standard matrices.
- Warning!
  - AB is not always equal to BA
  - $AB = AC$ does not mean that $B = C$
  - $AB = 0$ does not mean that A or B is 0

Find a 2×2 matrix A, not equal to I, with $A^4 = I$.

Summary of Section 3.5

- A is invertible if there is a matrix B (called the inverse) with

$$AB = BA = I_n$$

- For a 2×2 matrix A we have that A is invertible exactly when $\det(A) \neq 0$ and in this case

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

- If A is invertible, then $Ax = b$ has exactly one solution: $x = A^{-1}b$.
- $(A^{-1})^{-1} = A$ and $(AB)^{-1} = B^{-1}A^{-1}$
- Recipe for finding inverse: row reduce $(A \,|\, I_n)$.
- Invertible linear transformations correspond to invertible matrices.

Find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & h \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Summary of Section 3.6

- Say $A = n \times n$ matrix and $T : \mathbb{R}^n \to \mathbb{R}^n$ is the associated linear transformation. The following are equivalent.
  (1) A is invertible
  (2) T is invertible
  (3) The reduced row echelon form of A is I_n
  (4) etc.

In all questions, suppose that A is an $n \times n$ matrix and that $T : \mathbb{R}^n \to \mathbb{R}^n$ is the associated linear transformation.

(1) Suppose that the reduced row echelon form of A does not have any zero rows. Must it be true that $Ax = b$ is consistent for all b in $\mathbb{R}^n$?

YES NO

(2) Suppose that T is one-to-one. Is is possible that the columns of A add up to zero?

YES NO

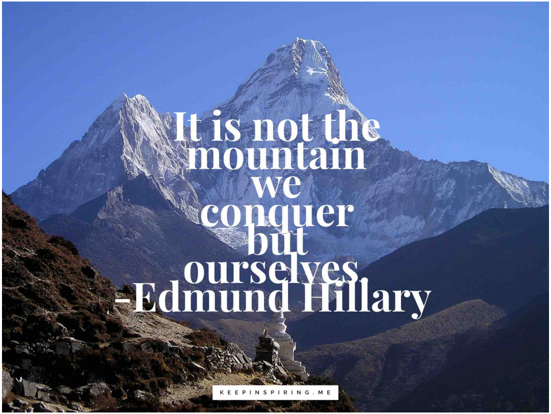
(3) Suppose that $Ax = e_1$ is not consistent. Is it possible that T is onto?

YES NO

Important terms

- subspace
- column space
- null space
- basis
- dimension
- one-to-one
- onto
- linear transformation
- inverse
- Rank theorem

Good luck!



It is not the mountain we conquer but ourselves.
-Edmund Hillary
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