Scores: 1 2 3 4 5 6 7 8 9 10 E Name Prof. M

Mathematics 2602

Section L1
Midterm 3
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16 November 2011

1. State the definition of a graph.

A graph is a nonempty set V together with a set E consisting of pairs of distinct elements of V.

State the definition of a graph isomorphism.

An isomorphism between two graphs is a bijection f from one vertex set to the other so that f and f^{-1} preserve edges.

2. Answer each of the following.

A graph has degree sequence 5, 5, 4, 4, 3, 3, 3, 3. How many edges does it have?

15

True or false: As we apply the Floyd–Warshall algorithm, the value of a particular matrix entry sometimes increases and sometimes decreases.

F

Suppose a plane graph has 10 vertices and 13 edges. How many regions does it divide the plane into?

5

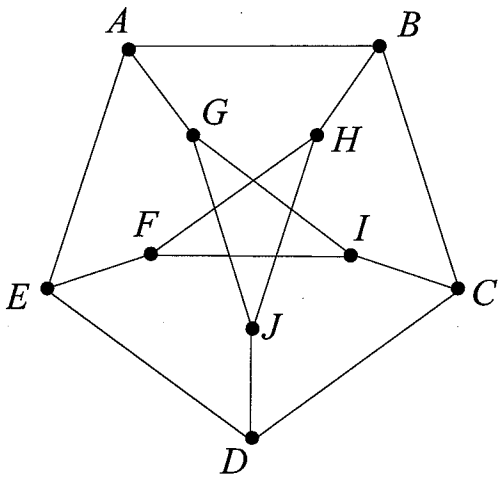
True or false: K_4 is isomorphic to a subgraph of $K_{4,4}$.

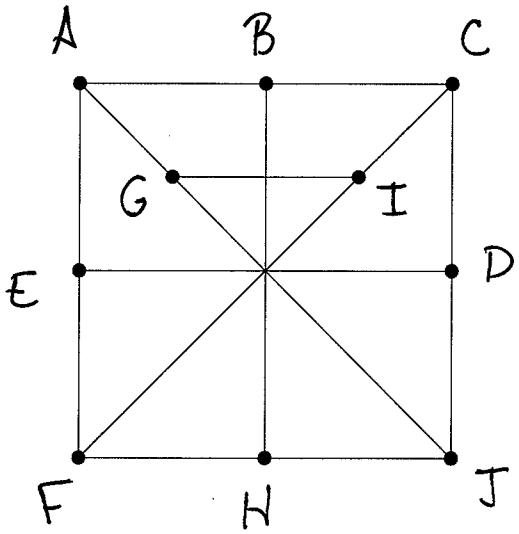
F

How many Platonic solids are there? Name one.

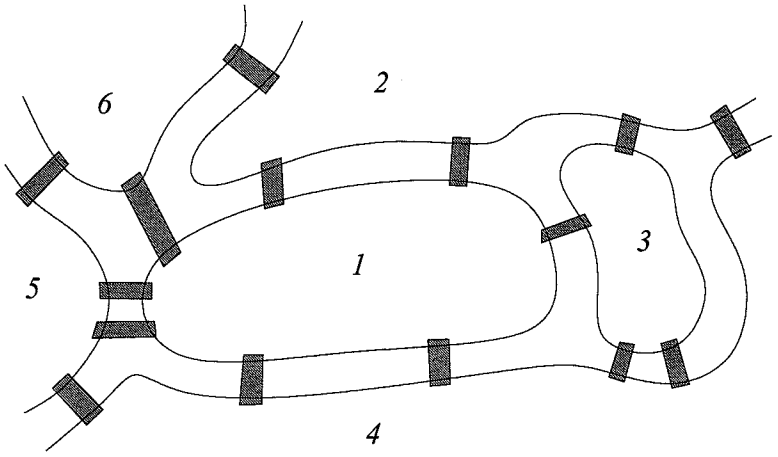
5 Cube

3. Show that the following two graphs are isomorphic by giving an explicit isomorphism. You can describe your isomorphism by labeling the vertices of the graph on the right by the corresponding labels from the graph on the left.



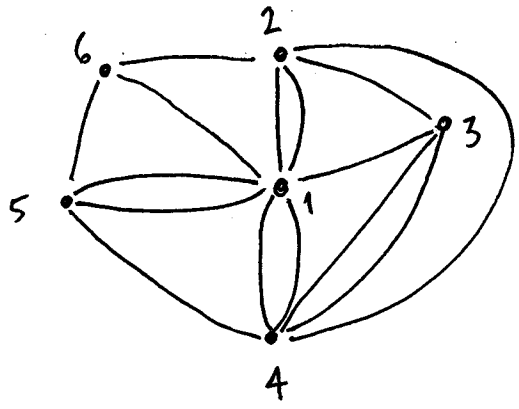


4. A city has the following map, with land masses labeled 1 through 6.



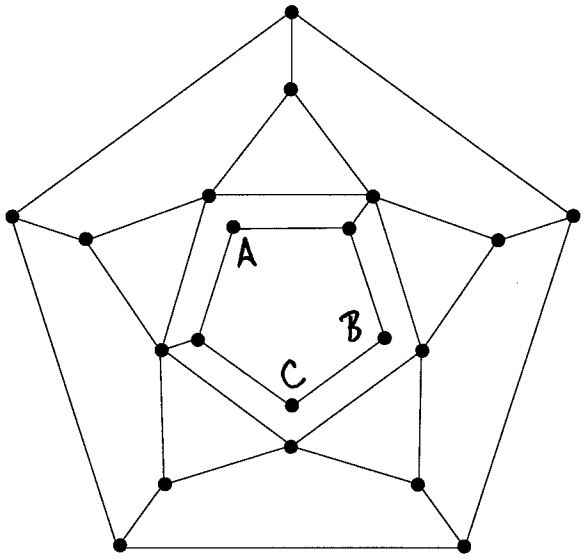
Is it possible for someone to take a walk, cross each bridge exactly once, and return to the starting point? Why or why not?

Such a walk would be an Eulerian Circuit in the pseudograph.



Since there is a vertex of odd degree (6 or 2) there is no such circuit.

5. Decide whether or not the following graph is Hamiltonian. If it is, exhibit a Hamiltonian cycle. If it is not, explain why not.



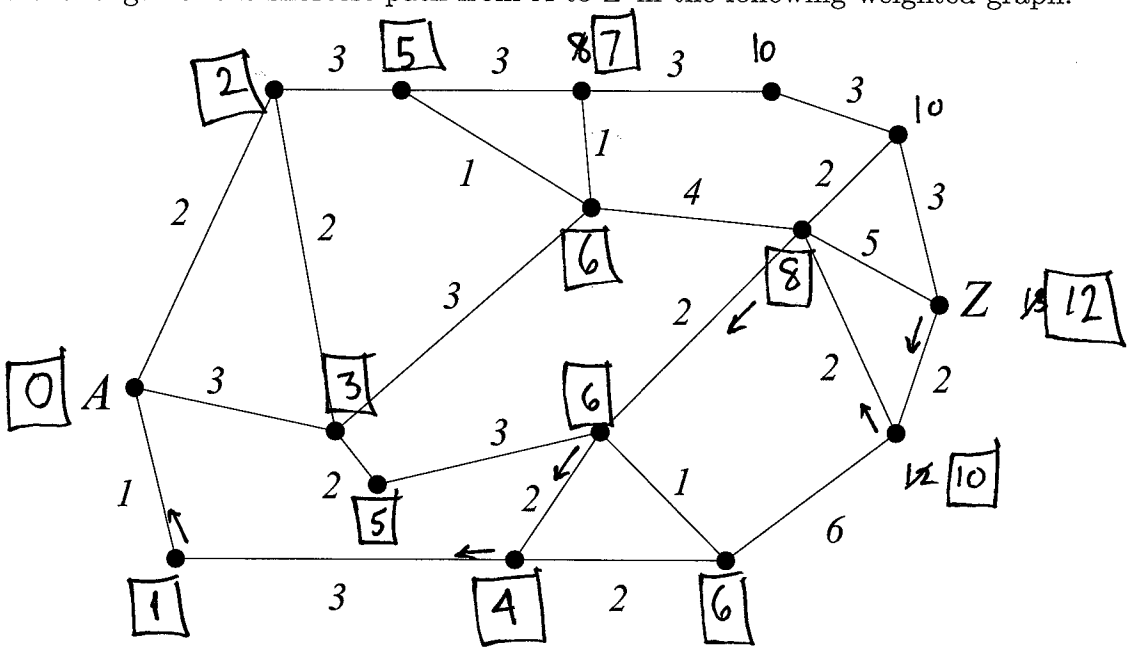
Since A, B, and C have degree 2, all edges incident to these must be in any Hamiltonian cycle.

So the entire inner pentagon must be in any Hamiltonian cycle.

But this is already a cycle that does not contain all vertices.

Cycles never contain proper subcycles, so there can't be a Hamiltonian cycle.

6. Find the length of the shortest path from A to Z in the following weighted graph.



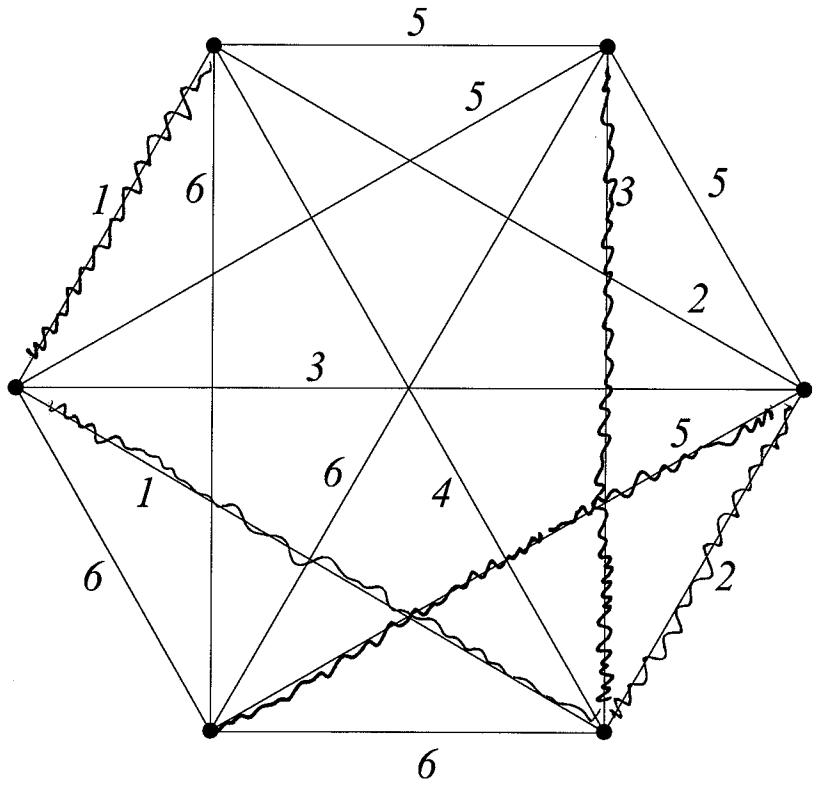
By Dijkstra's Algorithm:

12

How many paths of this length are there?

1

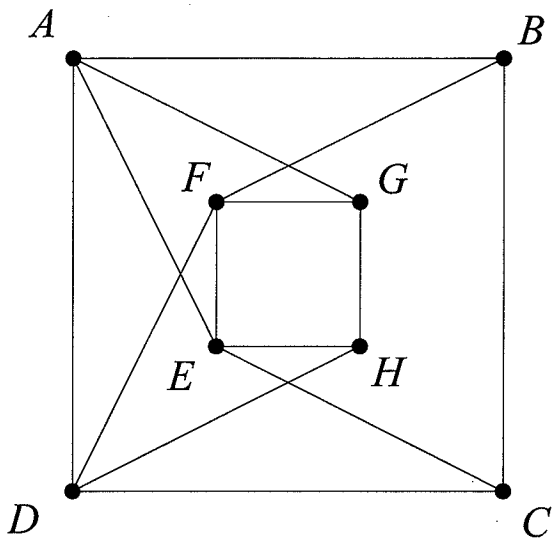
7. Find a minimal spanning tree for the following weighted graph. What is its weight?



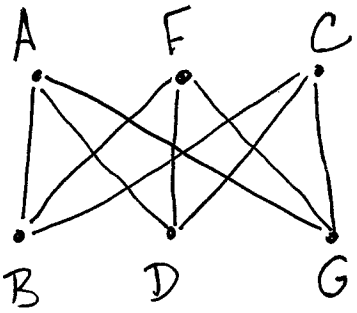
By Krushkal's algorithm:

$1+1+2+3+5=12$

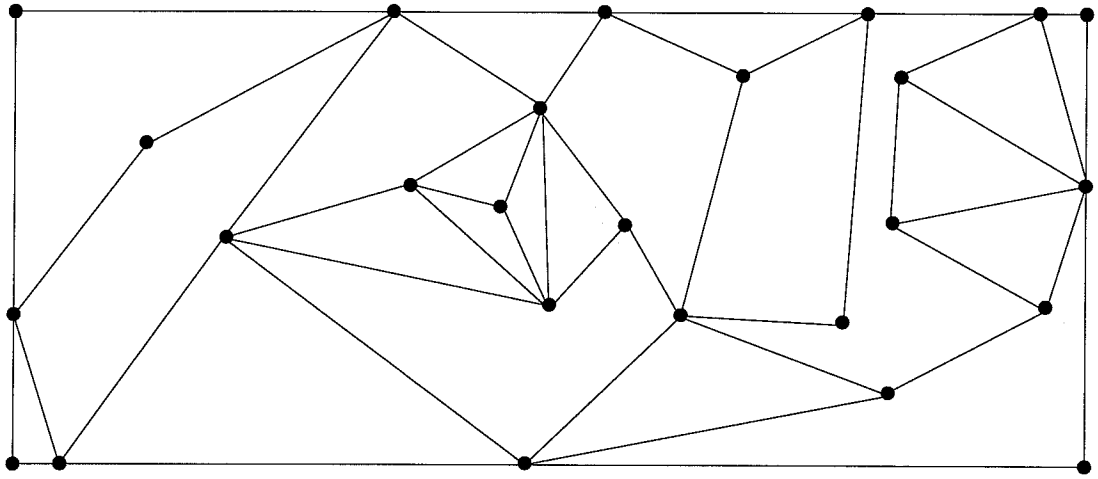
8. Is the following graph planar? Explain your answer.



No. It contains a subgraph isomorphic to $K_{3,3}$:



9. What is the chromatic number of the following graph? Explain your answer.

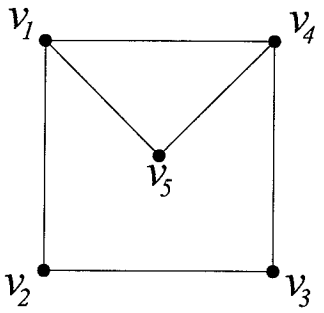


$\chi \leq 4$ by planarity

$\chi \geq 4$ since K_4 is a subgraph

So $\chi = 4$.

10. Suppose that we want to find the number of spanning trees for the following graph.



Kirchhoff's theorem says that we should find the cofactor of a certain matrix. What is that matrix for this graph?

$$\begin{pmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}$$

How many spanning trees does the graph have?

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Either take the determinant of $\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

or count the spanning trees by hand.

Extra credit (5 points). Explain why trees are bipartite graphs.

Let T be a tree.

Choose a vertex v_0, color it black.

For any other vertex v, color it black if $d(v_0, v)$ is even, and white otherwise.

By the uniqueness of paths in trees, adjacent vertices must have distances from v_0 that differ by 1.

Thus, black vertices are only connected to white and vice versa.

This shows that T is bipartite.