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Mathematics 2602

Quiz 10 (Extra Credit)

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1. Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the matrix transformation defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Compute the range of f . In other words, find an equation relating a , b , and c so that we can always compute values of x , y , and z for which

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Solve the linear system:

$$\begin{pmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ -1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 1 & 2 & 3 & a \\ -3 & -2 & -1 & b \\ -1 & 2 & 5 & c \end{pmatrix} \xrightarrow{\begin{matrix} \textcircled{2} + 3\textcircled{1} \\ \textcircled{3} + \textcircled{1} \end{matrix}} \begin{pmatrix} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 4 & 8 & a+c \end{pmatrix}$$

$$\xrightarrow{\textcircled{3} - \textcircled{2}} \begin{pmatrix} 1 & 2 & 3 & a \\ 0 & 4 & 8 & b+3a \\ 0 & 0 & 0 & c-b-2a \end{pmatrix}$$

For (1) to have solutions, we must have

$$c - b - 2a = 0$$

$$\iff \boxed{2a + b = c}$$