

Mathematics 2602

Quiz 11

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1. Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix}$$

The characteristic polynomial of A is

$$\det(\lambda I - A) = (\lambda - 1)(\lambda + 2) - 1 \cdot 4 = \lambda^2 + \lambda - 6$$

So the two eigenvalues are 2 and -3 .

For $\lambda = 2$, the matrix $\lambda I - A$ is

$$\begin{pmatrix} 1 & -4 \\ -1 & 4 \end{pmatrix}$$

Solving

$$\left(\begin{array}{cc|c} 1 & -4 & 0 \\ -1 & 4 & 0 \end{array} \right)$$

gives an eigenvector associated to $\lambda = 2$:

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

For $\lambda = -3$, the matrix $\lambda I - A$ is

$$\begin{pmatrix} -4 & -4 \\ -1 & -1 \end{pmatrix}$$

Solving

$$\left(\begin{array}{cc|c} -4 & -4 & 0 \\ -1 & -1 & 0 \end{array} \right)$$

gives an eigenvector associated to $\lambda = 3$:

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$