Name ________________________

Mathematics 2602

Quiz 1

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1. Prove by induction that $5^{2n} - 2^{5n}$ is divisible by 7 for $n \geq 0$.

First we check the base case, $n = 0$:

$$5^{2\cdot 0} - 2^{5\cdot 0} = 1 - 1 = 0.$$

Since 0 is divisible by 7, we are done with the base case.

Now assume that the statement is true for $n = k$. That is, assume

$$5^{2k} - 2^{5k}$$

is divisible by 7. Say that

$$5^{2k} - 2^{5k} = 7N$$

We now need to check that

$$5^{2(k+1)} - 2^{5(k+1)}$$

is divisible by 7. We have:

$$\begin{aligned}
5^{2(k+1)} - 2^{5(k+1)} &= 5^{2k+2} - 2^{5k+5} \\
&= 5^{2k}5^2 - 2^{5k}2^5 \\
&= 25 \cdot 5^{2k} - 32 \cdot 2^{5k} \\
&= 32 \cdot 5^{2k} - 32 \cdot 2^{5k} - 7 \cdot 5^{2k} \\
&= 32(5^{2k} - 2^{5k}) - 7 \cdot 5^{2k} \\
&= 32(7N) - 7 \cdot 5^{2k} \\
&= 7(32N - 5^{2k}).
\end{aligned}$$

This is divisible by 7, as desired.

By the principle of mathematical induction, $5^{2n} - 2^{5n}$ is divisible by 7 for $n \geq 0$.