Name Answer Key

Mathematics 2602

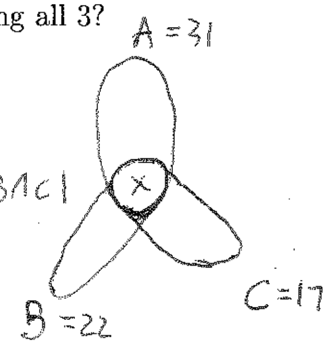
Quiz 3
Prof. Margalit
21 September 2011

1. A professor makes a multiple choice quiz with 5 questions. The first 3 questions have 5 possible answers each, and the last 2 questions have 4 possible answers each. How many different answer keys can there possibly be? Do not simplify your answer.

$5 \cdot 5 \cdot 5 \cdot 4 \cdot 4 = \boxed{5^3 4^2}$ $(=2000)$

5 choices each for the first 3   4 choices each for the last 2

2. There are 60 total students in Algebra, Biology, and Chemistry this semester. There are 31 students taking Algebra, 22 taking Biology, and 17 taking Chemistry. If none of the students are taking exactly 2 of these classes, what is the number of students taking all 3?



Let x be the number of students taking all 3.

$$|A \cup B \cup C| = 60 = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 31 + 22 + 17 - x - x - x + x \Rightarrow \underline{60 = 70 - 2x}$$

* Note that $|A \cap B| = |A \cap B \cap C|$, since no one takes exactly 2, if one takes 2 classes he/she must take the 3rd one.

$\boxed{x = 5}$

3. Suppose there are 25 students in the class born in 1991. Can we say with certainty that at least 3 of these 25 students were born in the same month? Why or why not?

$\boxed{\text{Yes}}$ By the pigeonhole principle (6.3.2)

$n = 25$ and $m = 12$ (there are 12 months in a year), so there must be some month with $\lceil \frac{25}{12} \rceil = 3$ people were born in.

You can also argue that in the worst case, 24 students can be distributed equally, so that each month has 2 students, but then 25^{th} students forces that at least 3 students to be born in the same month. □