Review

1. A polygon is convex if the line segment connecting any two vertices of the polygon is completely contained in the polygon.

Prove that the sum of the interior angles of a convex n-gon is $(n-2)\pi$.

2. A finite collection of lines in the plane determines a "map." Show that this map can be colored with 2 colors.

3. Define a sequence a_n by $a_0 = 0$, $a_n = a_{\lfloor \frac{n}{5} \rfloor} + a_{\lfloor \frac{3n}{5} \rfloor} + n$

What is a_{10}?

Prove that $a_n \leq 20n$ for all n.

4. Let a_n denote the number of n-digit numbers, each of whose digits is 1, 2, 3, or 4 and in which the number of 1's is even.

What is a_3? Find a recursive formula for a_n.

5. Solve the recurrence relation $a_n = 4a_{n-1} + 8^n$, where $a_0 = 1$.

6. What is the generating function associated to the sequence $2, 6, 18, 54, \ldots$?

7. Use generating functions to solve $a_n = 3a_{n-1} + 1$, $a_0 = 1$.

8. How many ways are there to walk up a flight of n stairs if you can go up either 1 or 2 steps at a time?

9. True or false: If f is $O(g)$ and h is $O(g)$ then fh is $O(g)$.

10. True or false: $\log_2 n$ and $\log_3 n$ have the same order.

11. Say x is a real number and n a natural number. Here are two algorithms for computing x^{2^n}. Which is more efficient (in terms of number of multiplications)?

A. Set $a = 1$
For $i = 1$ to 2^n, replace
a by xa
Output a

B. Set $a = x$.
For $i = 1$ to n, replace
a by a^2
Output a.

12. Show that $n \log n < n^2$.

13. Which of the following are equal?
    A. The number of 2-element subsets of $\{1,\ldots,n\}$
    B. The number of edges in K_n
    C. $1+2+\cdots+(n-1)$
    D. The number of ways of putting 2 marbles in n boxes, any number to a box.

14. In a room with 19 cats, there are twice as many dumb cats as ugly cats. The number of cats that are neither dumb nor ugly is twice the number of cats that are dumb and ugly. There is one ugly cat who is not dumb. How many cats are dumb but not ugly?

15. Write down any list of 6 natural numbers. There must be a string of consecutive numbers with sum divisible by... ?

16. How many poker hands contain exactly 3 royal cards?

17. Ten students form 3 study groups, each with at least two people. How many ways are there to do this?

18. Bridge hands are dealt to Alice, Bob, Charley, and Daisy. What is the probability that Alice has 3 red cards given that she and her partner got 9 red cards?

19. For which n is K_n isomorphic to C_n?

20. For which m,n,k is $K_{m,n}$ isomorphic to C_k?

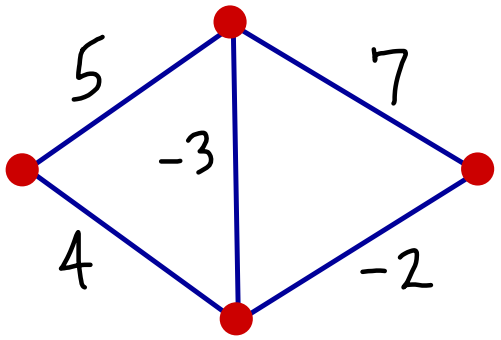
21. For which m,n is $K_{m,n}$ Eulerian? Hamiltonian?

22. Is it possible for a knight to visit every square of a chessboard and return to where it started?

What about a 7×7 board?

23. How many spanning trees does K_4 have? K_5?

24. Find the "distances" between all pairs of vertices using the Floyd-Warshall algorithm.

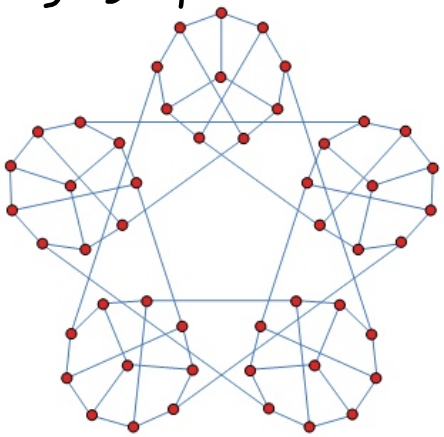


25. How many isomers of C_7H_{16} are there?

26. How many connected graphs have chromatic number 1?

27. The cone on a graph G is obtained by adding one new vertex and connecting it by edges to all old vertices. What is the chromatic number of the cone on $K_{3,2}$? What about the cone on C_n?

28. Is the following graph planar?



29. Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$, $a_0 = 0$, $a_1 = 1$ using linear algebra.

30. Maximize $2x_1 + 4x_2 + 3x_3 + x_4$

subject to $3x_1 + x_2 + x_3 + 4x_4 \leq 12$

$x_1 - 3x_2 + 2x_3 + 3x_4 \leq 7$

$2x_1 + x_2 + 3x_3 - x_4 \leq 10$

$x_i \geq 0$