Scores: 1-4 5 6 7 8 9 10 Name ______________________

Section L___

Mathematics 2602

Midterm 3
Prof. Margalit
5 April 2012

1. Answer the following questions with your clicker, and record your answers on this page.

$\boxed{\text{Clicker \#1}}$ What is the definition of a graph?

A. A graph is a pair of sets V and E, where V is nonempty, and each element of E is a pair of elements of V.

B. A graph is a pair of sets V and E, where V is nonempty, and each element of E is an ordered pair of elements of V.

C. A graph is a pair of sets V and E, where V is nonempty, and each pair of elements of V is an element of E.

D. A graph is a collection of vertices, together with a collection of edges, each connecting two vertices.

$\boxed{\text{Clicker \#2}}$ Which of the following are characterizations of a tree? Choose all that apply.

A. A tree is a connected graph with no circuits.

B. A tree is a connected graph with n vertices and $n-1$ edges.

C. A tree is a connected graph with no cycles.

D. A tree is a graph with no circuits or cycles.

2. Answer the following questions with your clicker, and record your answers on this page.

$\boxed{\text{Clicker \#3}}$ Which of the following pairs of sets define graphs? Select all that apply.

A. $V = \{v\}$, $E = \emptyset$

B. $V = \{u, v, w\}$, $E = \{\{u, v\}, \{v, w\}\}$

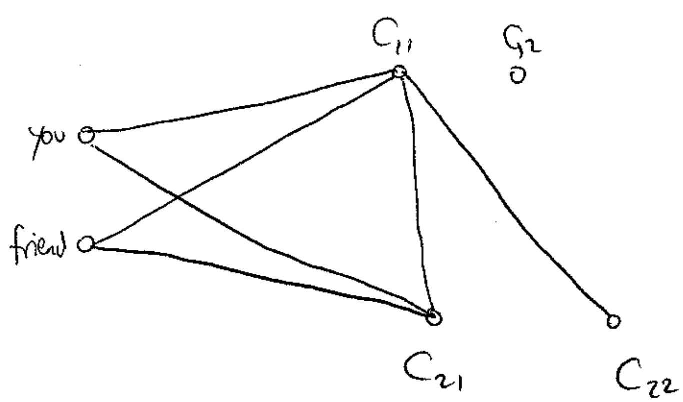
C. $V = \emptyset$, $E = \emptyset$

D. $V = \{u, v\}$, $E = \{\{u, v\}, \{v, w\}\}$

$\boxed{\text{Clicker \#4}}$ True or false: The cycle C_{101} is a bipartite graph.

false

$\boxed{\text{Clicker \#5}}$ You and a friend meet two other couples at a party. Nobody hugs themselves, there are no hugs within couples, and no one hugs the same person more than once. The number people hugged by the other five people (excluding you) are all different. How many people did you hug?



you hug
2 people.

3. Answer the following questions with your clicker, and record your answers on this page.

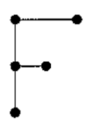
$\boxed{\text{Clicker \#6}}$ A tree has 10 edges. How many vertices does it have?

11

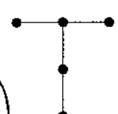
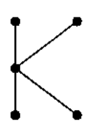
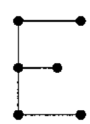
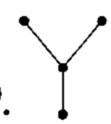
$\boxed{\text{Clicker \#7}}$ If a graph has degree sequence $3, 3, 2, 2, 2$, how many edges does it have?

$$\frac{3+3+2+2+2}{2} = \frac{12}{2} = 6.$$

$\boxed{\text{Clicker \#8}}$ Consider the following graph.



Which of the following graphs are isomorphic to the above graph? Select all that apply.

A.  B.  C.  D. 

4. Answer the following questions with your clicker, and record your answers on this page.

$\boxed{\text{Clicker \#9}}$ If a planar graph with 10 vertices divides the plane into 3 regions, how many edges does it have?

$V = 10$, $F = 3$

$V - E + F = 2$

$\Rightarrow E = V + F - 2 = 10 + 3 - 2 = 11$

$\boxed{\text{Clicker \#10}}$ True or false: Every planar graph has chromatic number 4.

false

(every planar graph has chromatic # ≤ 4)

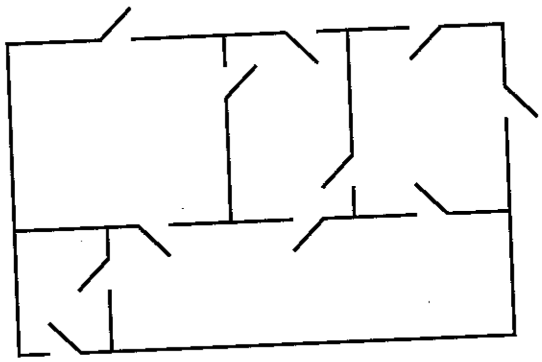
$\boxed{\text{Clicker \#11}}$ Suppose the 3rd matrix obtained via the Floyd–Warshall algorithm is

$$\begin{pmatrix} 0 & 2 & 5 & 4 \\ 2 & 0 & 6 & 2 \\ 5 & 6 & 0 & 1 \\ 4 & 2 & 1 & 0 \end{pmatrix}$$

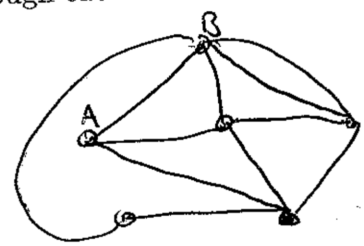
What is the $(2,3)$-entry of the 4th matrix? (Note: the very first matrix in the Floyd–Warshall algorithm is the 0th matrix.)

$$M_4(2,3) = \min\{M_3(2,3), M_3(2,4) + M_3(4,3)\}$$
$$= \min\{6, 2+1\}$$
$$= \min\{6, 3\}$$
$$= 3$$

5. Consider the following floor plan of a house.



Explain why it is not possible to take a tour of the house, passing through each door exactly once, and returning to where you started.



It would be an Eulerian circuit of

and A has deg 3, so that

graph has no Eulerian circuit.

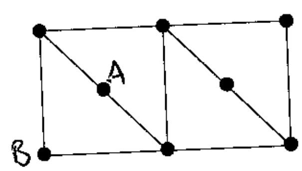
Is it possible to add a single door so that such a tour is possible? Why or why not?

Yes. Add a door that corresponds to an edge between A & B
(ie. top-left room to outside)

This would make all vertices have even degree

$\Rightarrow$ Eulerian circuit would exist.

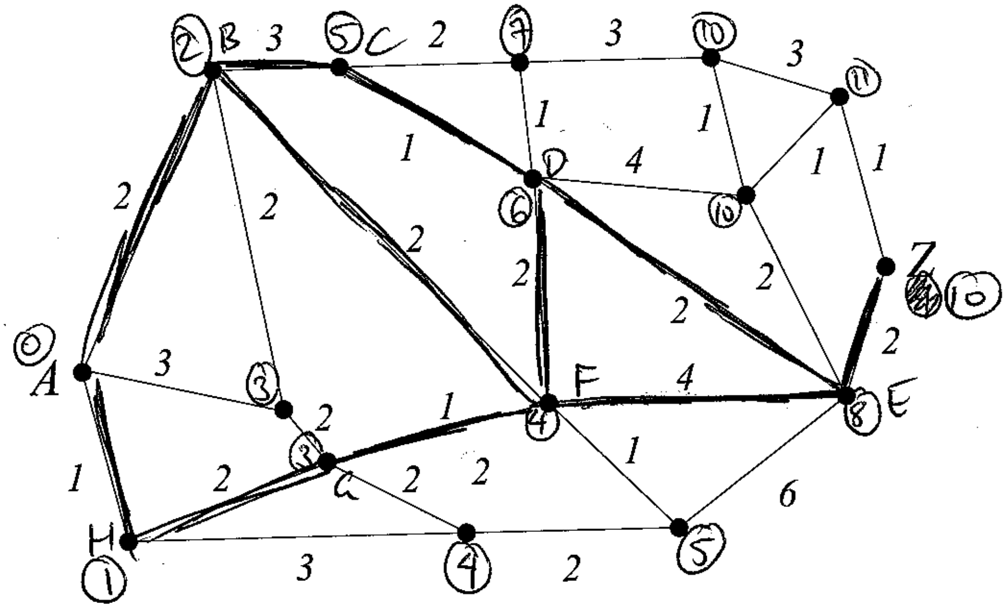
Is the following graph Hamiltonian? Explain your answer.



Since $\deg A = \deg B = 2$, both edges incident to each vertex must be part of the Hamiltonian cycle. but this would make a cycle inside the Hamiltonian cycle, which is not allowed.

So no, there is no Hamiltonian cycle.

6. Consider the following graph.



What is the distance from A to Z?

10.

How many paths are there from A to Z that have the shortest possible distance?

there are 5 ~~4~~ routes:

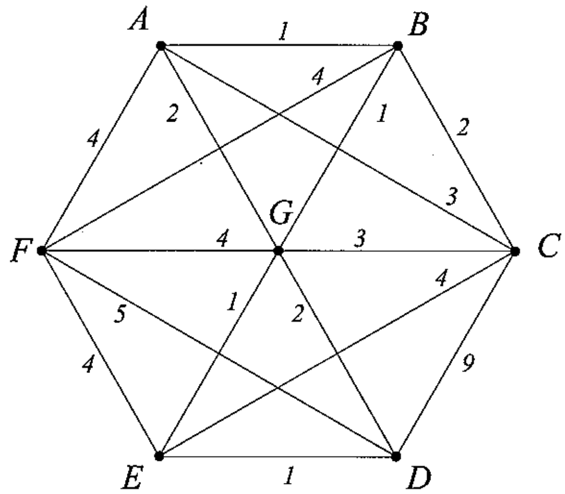
ABCDEZ, ABFEZ, ABFDEZ, AHGFEZ,

and AHGFDEZ

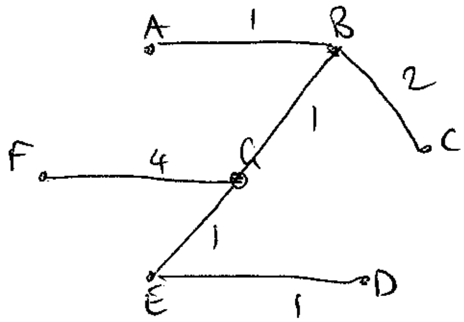
How many vertices have distance 10 from A?

3

7. Consider the following graph.



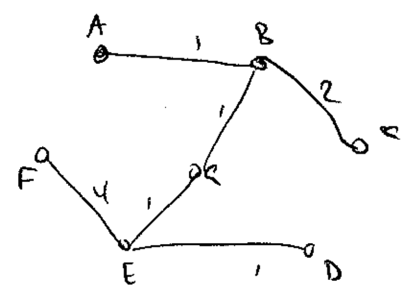
What is the weight of a minimal spanning tree for this graph?



weight = 10.

If you use Prim's algorithm, starting from vertex A, what is the order in which you reach the seven vertices?

A , B , G , E , D , C , F

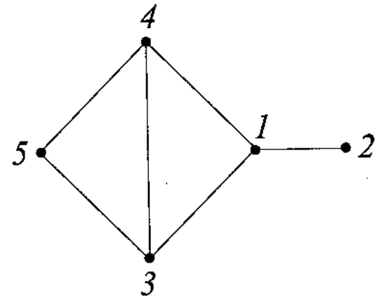


If you use Krushkal's algorithm, how many different minimal spanning trees can you obtain?

Can get all weight 1 edges
then only weight 2 edge to pick is BC.
then only need to connect F ; so 4~~8~~ choices
of edges of weight 4.

so 4 different spanning
trees of minimal weight

8. Consider the following graph.



Kirchhoff's theorem says that to compute the number of spanning trees we should first write down a 5×5 matrix. What is that matrix?

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 3 & -1 & -1 \\ -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{pmatrix} \end{array}$$

What is the number of spanning trees for the above graph? Either take the determinant of a 4×4 matrix, or give another explanation.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{vmatrix} = 8$$

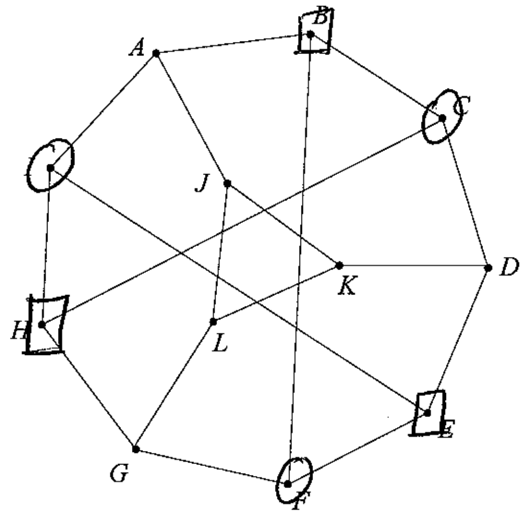
the edge (1,2) is req^d. then you either have 2 of the 3 edges in the Δ

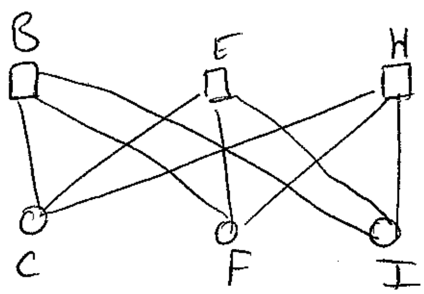
(in 3 ways) with one of the edges (3,5) or ~~edges~~ (4,5) (so total 6 ways)

or only one edge of the triangle, (1,4) or (1,3) together with both (3,5) & (4,5). and that in 2 ways

So total $6 + 2 = 8$ ways

9. Is the following graph planar? Explain your answer.





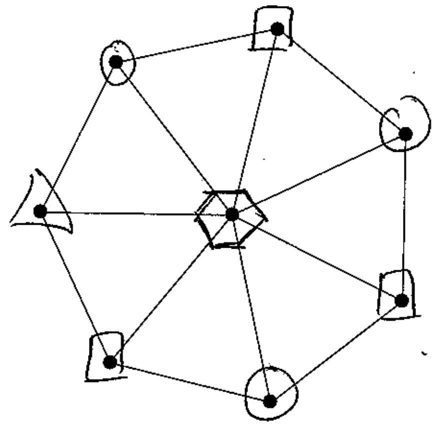
this is an embedded $K_{3,3}$
since ~~#~~ $K_{3,3}$ is not planar,
So the original is not planar.

Explain why every tree is planar.

A tree has no cycles, and therefore no subgraph can be isomorphic to $K_{3,3}$ or K_5, which both contain cycles.

Therefore, a tree is planar, by Kuratowski's theorem.

10. Find the chromatic number of the following graph.



this colouring shows $\chi(G) \leq 4$.

Since the "outside" cycle of G is an odd cycle, it requires 3 colours. the "centre" vertex is connected to all vertices in the cycle, hence needs a fourth colour. So $\chi(G) \geq 4$

$\Rightarrow \chi(G) = 4$.

Suppose that G is a tree. What is $\chi(G)$? Explain your answer.

Choose a starting vertex v and colour it blue.
Every other vertex is either even or odd distance away from v, as G being a tree $\Rightarrow$ there is a unique path from v to everywhere. Thus for any other vertex, colour it blue if it is even distance away from v, and red if it is odd distance away from v. This is a 2-colouring of G.
Thus if G has ≥ 2 vertices, then $\chi(G) = 2$
Otherwise if $G = K_1$, then $\chi(G) = 1$.