Name Solutions

Mathematics 2602

Quiz 10
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18 April 2012

Consider the matrix

$$A = \begin{pmatrix} -1 & 0 & 0 \\ -6 & 3 & 2 \\ 6 & -4 & -3 \end{pmatrix}$$

Find the eigenvalues of A.

$$0 = |A - \lambda I_3| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ -6 & 3-\lambda & 2 \\ 6 & -4 & -3-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(3-\lambda)(-3-\lambda) - (-1-\lambda)(2)(-4)$$

$$= (-1-\lambda)(-9+\lambda^2+8) = -(\lambda+1)^2(\lambda-1)$$

So $\lambda = 1, -1$

For each eigenvalue λ of A, find all of the eigenvectors for λ.

$\underline{\lambda=-1}$: $(A + I_3 \mid \vec{0}) = \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -6 & 4 & 2 & 0 \\ 6 & -4 & -2 & 0 \end{array}\right)$

$$\downarrow$$

$$\begin{array}{c} \frac{1}{2}② \\ ① \\ ③+② \end{array} \left(\begin{array}{ccc|c} \overset{a}{-3} & \overset{b}{2} & \overset{c}{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

so $-3a + 2b + c = 0$

so $\vec{x} = \begin{pmatrix} \frac{2}{3}b + \frac{1}{3}c \\ b \\ c \end{pmatrix} = b\begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 1/3 \\ 0 \\ 1 \end{pmatrix}$ so $\begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/3 \\ 0 \\ 1 \end{pmatrix}$ are e'vectors

$\underline{\lambda=1}$: $(A - I_3 \mid \vec{0}) = \left(\begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ -6 & 2 & 2 & 0 \\ 6 & -4 & -4 & 0 \end{array}\right)$

$$\downarrow$$

$$\begin{array}{c} -\frac{1}{2}① \\ \frac{1}{2}(②-3①) \\ ③+2②-3① \end{array} \left(\begin{array}{ccc|c} \overset{a}{1} & \overset{b}{0} & \overset{c}{0} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

So $a = 0$, $b = -c$

$\vec{x} = \begin{pmatrix} 0 \\ -c \\ c \end{pmatrix} = c\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

So $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is e'vector

Extra credit. Is A diagonalizable?

Yes, as A has 3 linearly independent e'vectors.