Name Solution

Mathematics 2602
Quiz 1
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1. Use the principle of mathematical induction to prove that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for $n \geq 1$.

Proof by Induction:

The base case is $n=1$:

Left side $= 1^3 = 1$

right side $= \frac{(1)^2((1)+1)^2}{4} = 1$.

Induction Hypothesis: assume for some $k \geq 1$ that $1^3+2^3+\cdots+k^3 = \frac{k^2(k+1)^2}{4}$.

Induction Step: $\underbrace{1^3 + 2^3 + \cdots + k^3} + (k+1)^3$

$= \frac{k^2(k+1)^2}{4} + (k+1)^3$ by induction hypothesis

$= (k+1)^2\left(\frac{k^2}{4} + k+1\right)$

$= \frac{(k+1)^2}{4}(k^2+4k+4)$

$= \frac{(k+1)^2(k+2)^2}{4} = \frac{(k+1)^2((k+1)+1)^2}{4}$.

Thus by the principles of mathematical induction,

$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \geq 1$.