1. Solve the recurrence relation

\[ a_n = a_{n-1} + 2a_{n-2} \]

with initial conditions \( a_0 = 2 \) and \( a_1 = 1 \).

This is a second order homogeneous recurrence relation. Its characteristic polynomial is:

\[ x^2 - x - 2 = (x - 2)(x + 1), \]

whose roots are \( x = 2 \) and \( x = -1 \). Thus, the solution to the recurrence relation is of the form

\[ a_n = c_1 2^n + c_2 (-1)^n. \]

We now solve for \( c_1 \) and \( c_2 \). We have:

\[ a_0 = c_1 2^0 + c_2 (-1)^0 = c_1 + c_2 = 2 \]

and

\[ a_1 = c_1 2^1 + c_2 (-1)^1 = 2c_1 - c_2 = 1 \]

Solving this system of two equations, we find \( c_1 = 1 \) and \( c_2 = 1 \). Thus, we have:

\[ a_n = 2^n + (-1)^n. \]