Name ________________________

Mathematics 2602

Quiz 2

Prof. Margalit

7 September 2011

1. Solve the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2}$$

with initial conditions $a_0 = 2$ and $a_1 = 1$.

This is a second order homogeneous recurrence relation. Its characteristic polynomial is:

$$x^2 - x - 2 = (x-2)(x+1),$$

whose roots are $x = 2$ and $x = -1$. Thus, the solution to the recurrence relation is of the form

$$a_n = c_1 2^n + c_2(-1)^n.$$

We now solve for c_1 and c_2. We have:

$$a_0 = c_1 2^0 + c_2(-1)^0 = c_1 + c_2 = 2$$

and

$$a_1 = c_1 2^1 + c_2(-1)^1 = 2c_1 - c_2 = 1$$

Solving this system of two equations, we find $c_1 = 1$ and $c_2 = 1$. Thus, we have:

$$a_n = 2^n + (-1)^n.$$